New Diffie–Hellman Speed Records

Chitchanok Chuengsatiansup

Technische Universiteit Eindhoven

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Diffie–Hellman Key Exchange

a b

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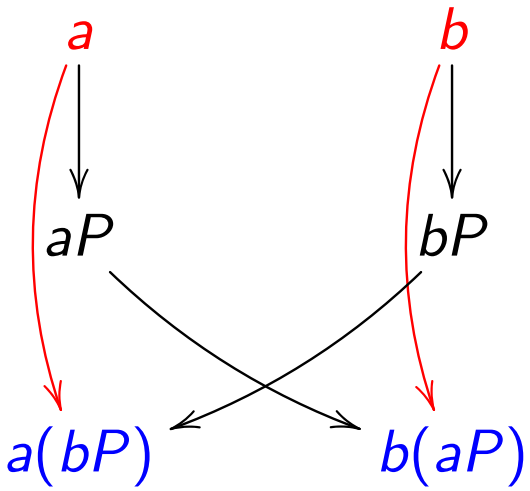

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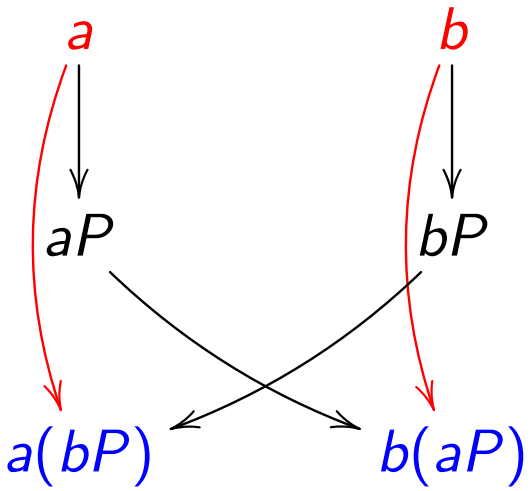

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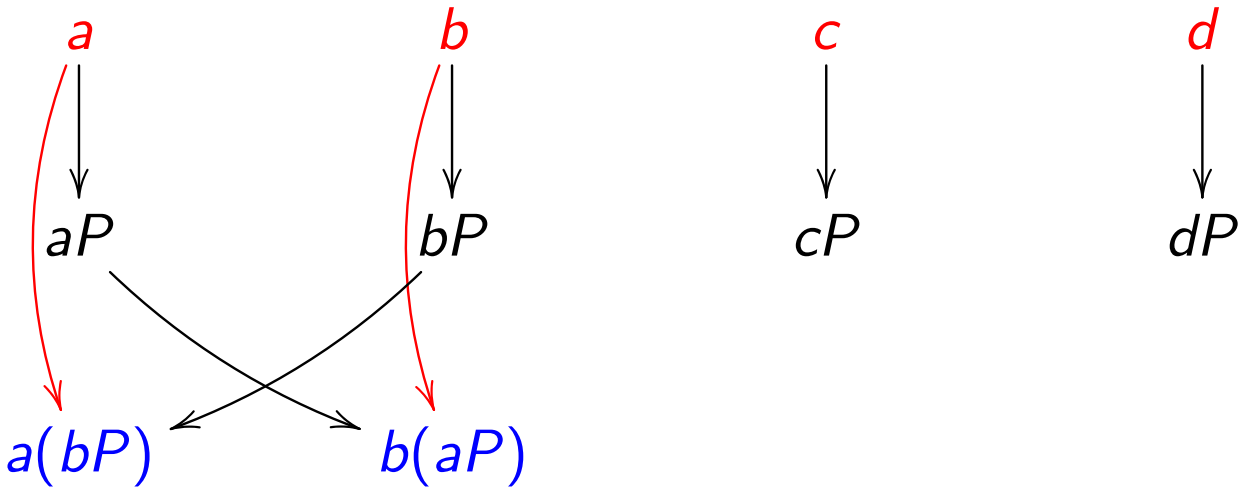

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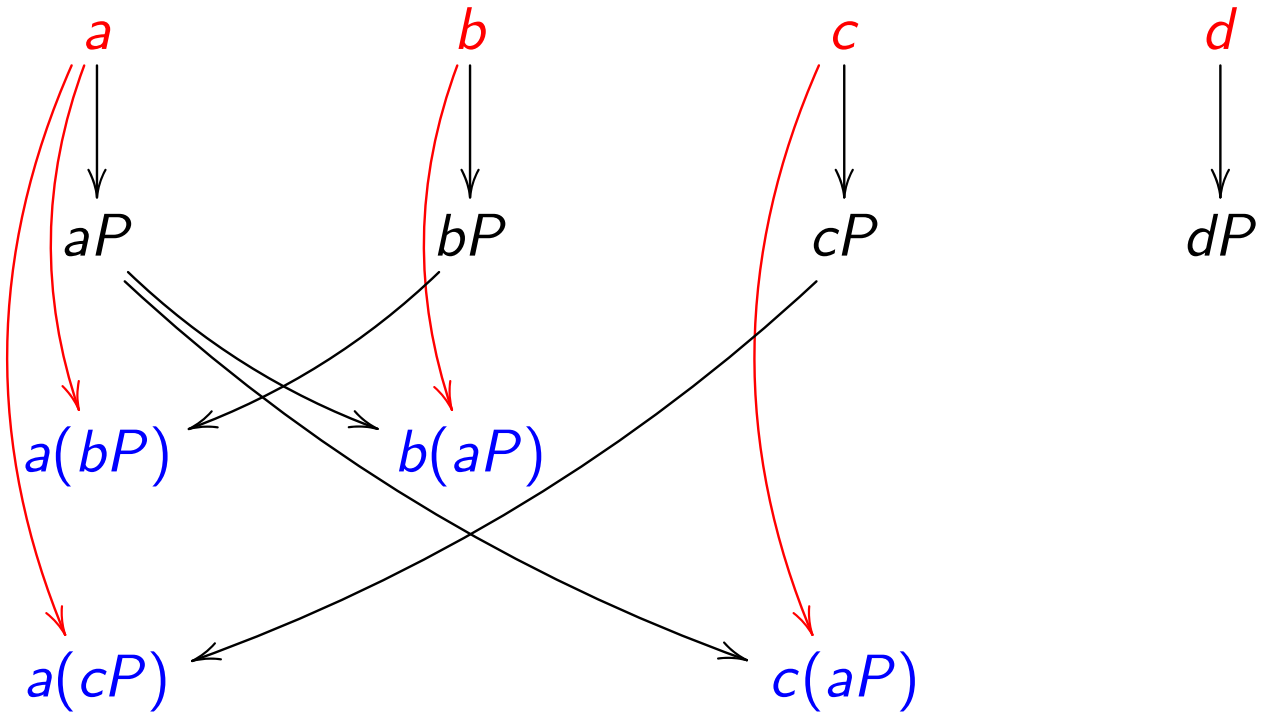

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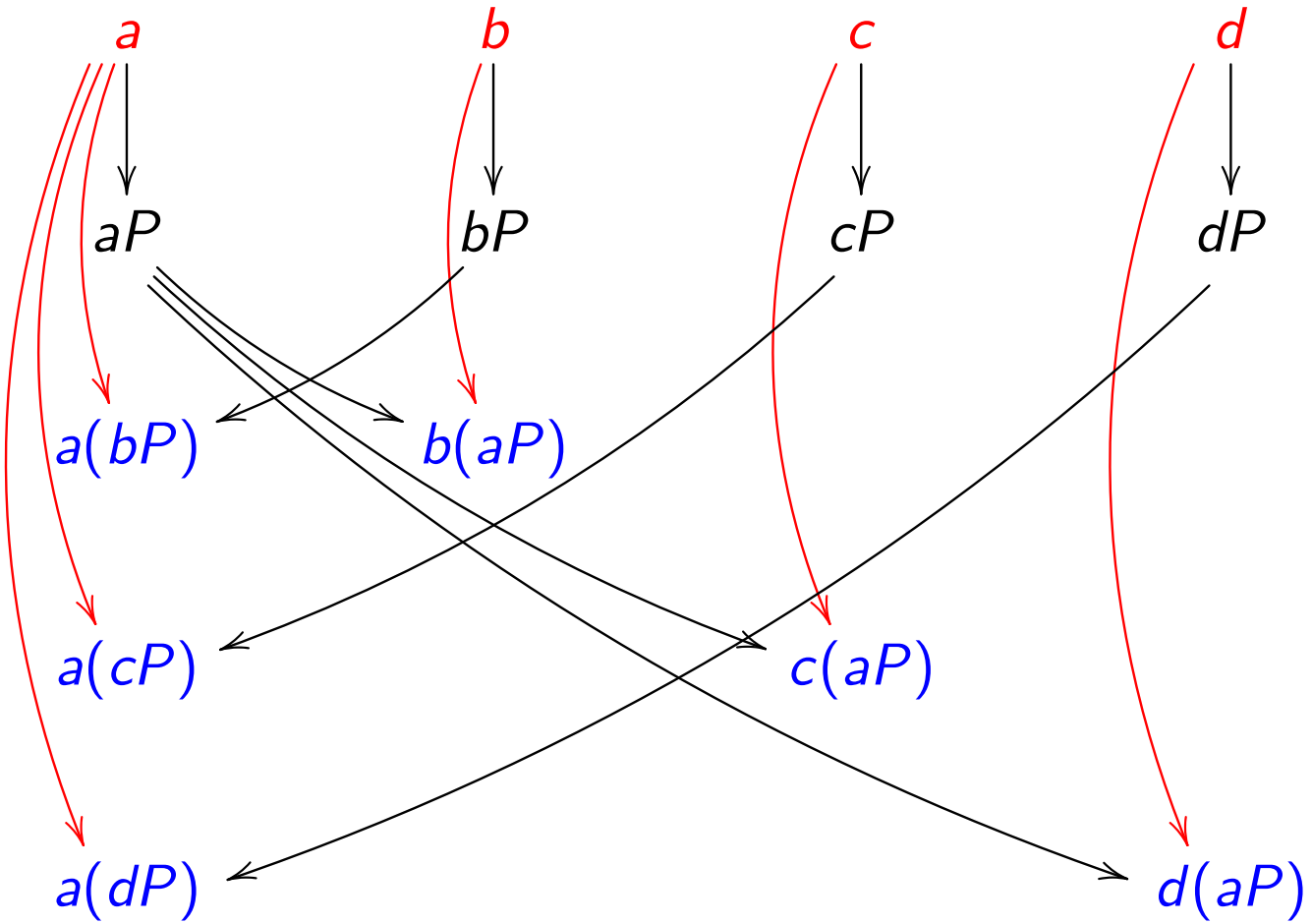

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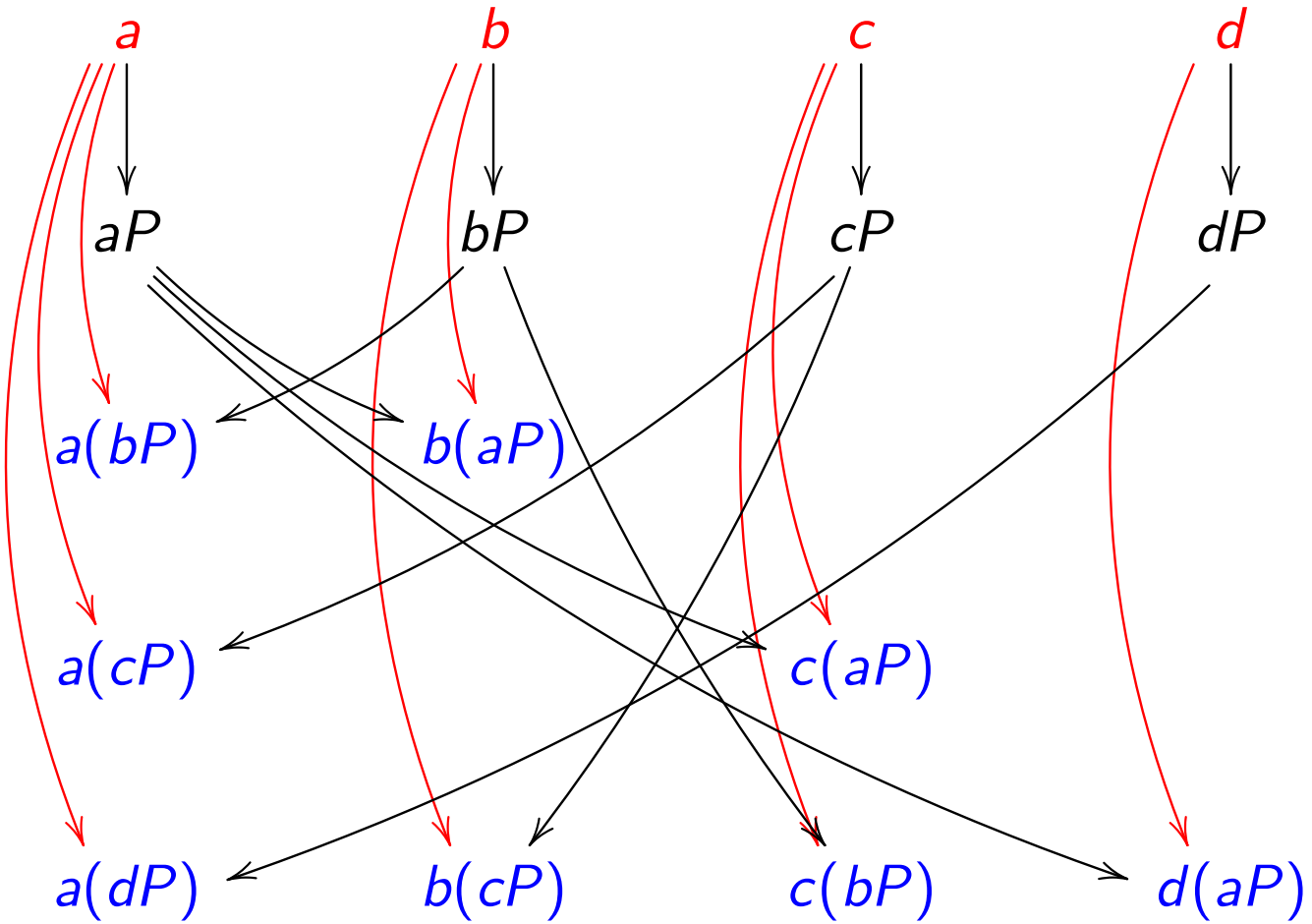

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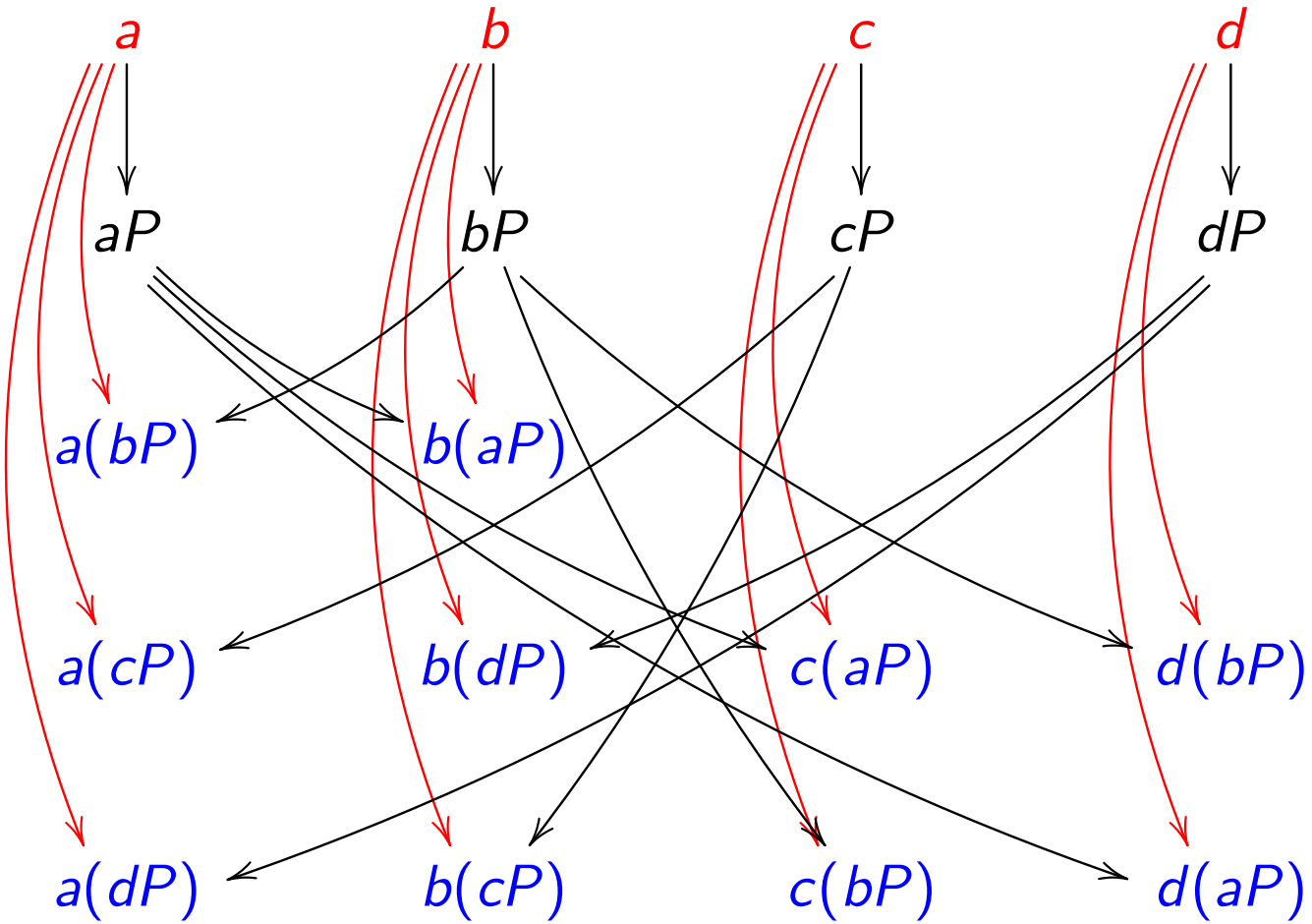

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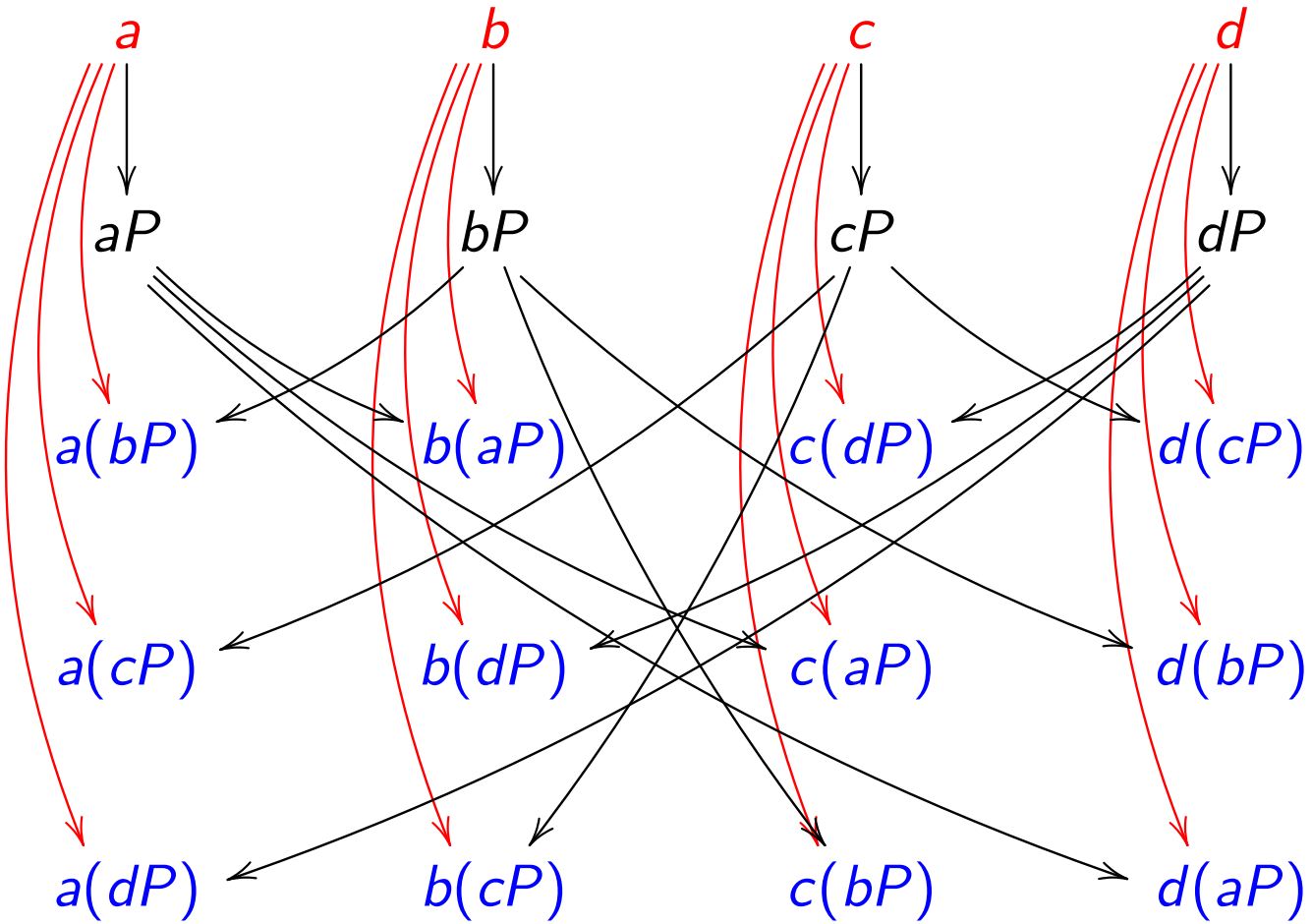

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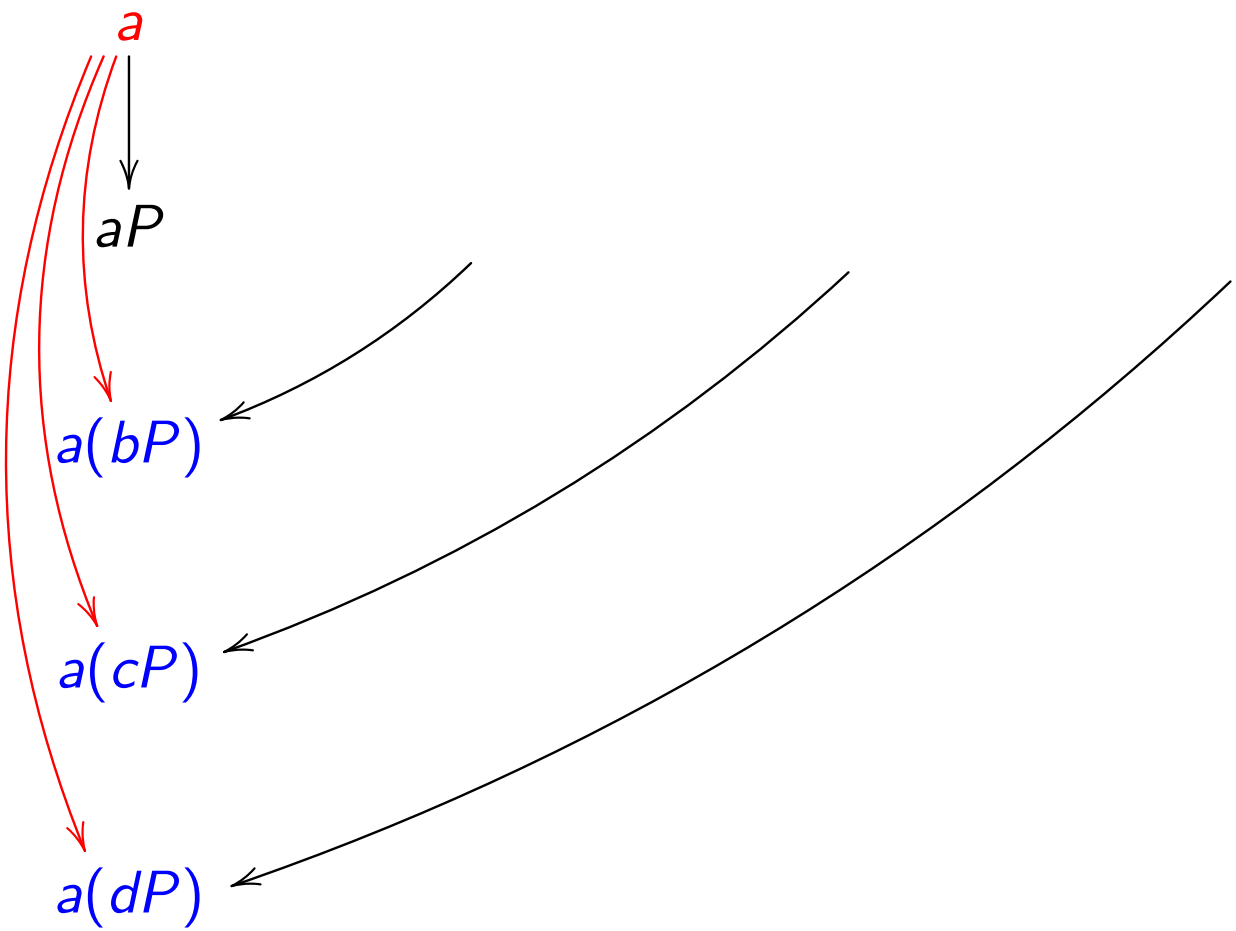

main DH challenge: make variable-base scalar mult as fast as possible

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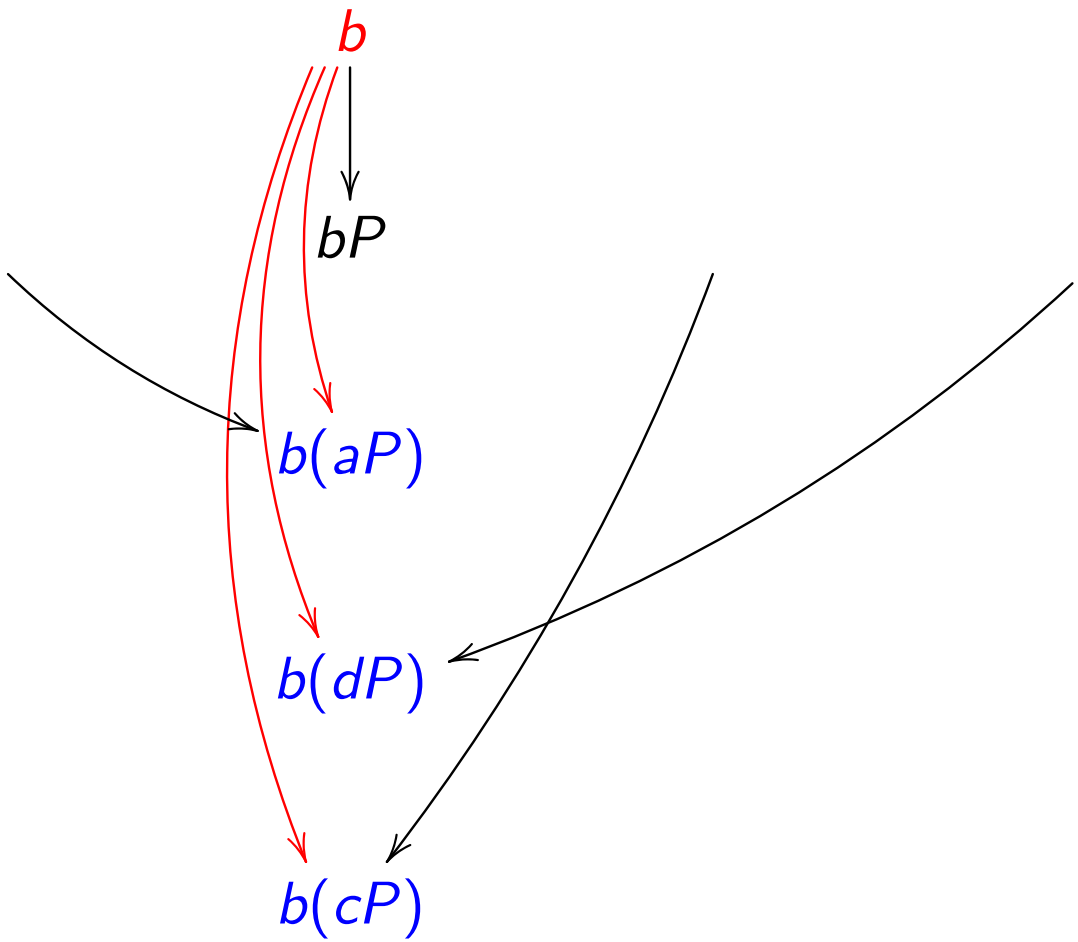

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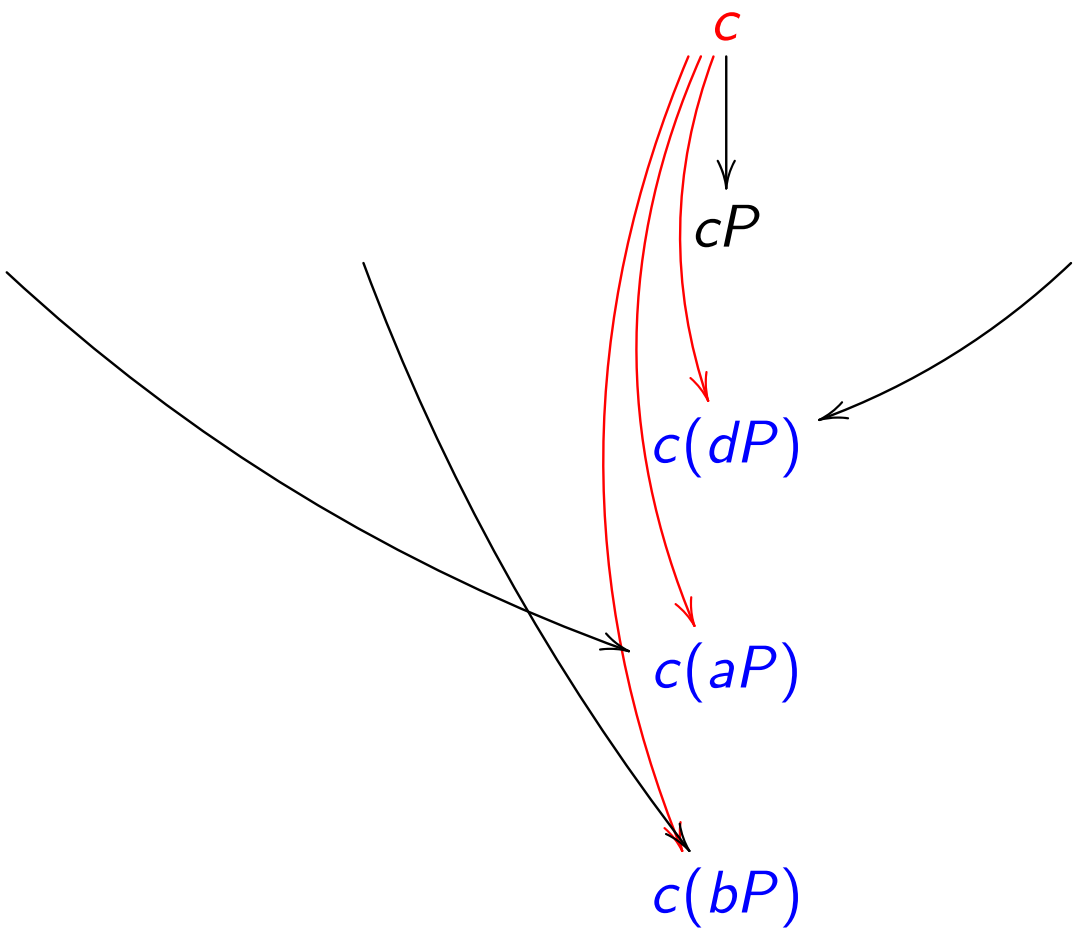

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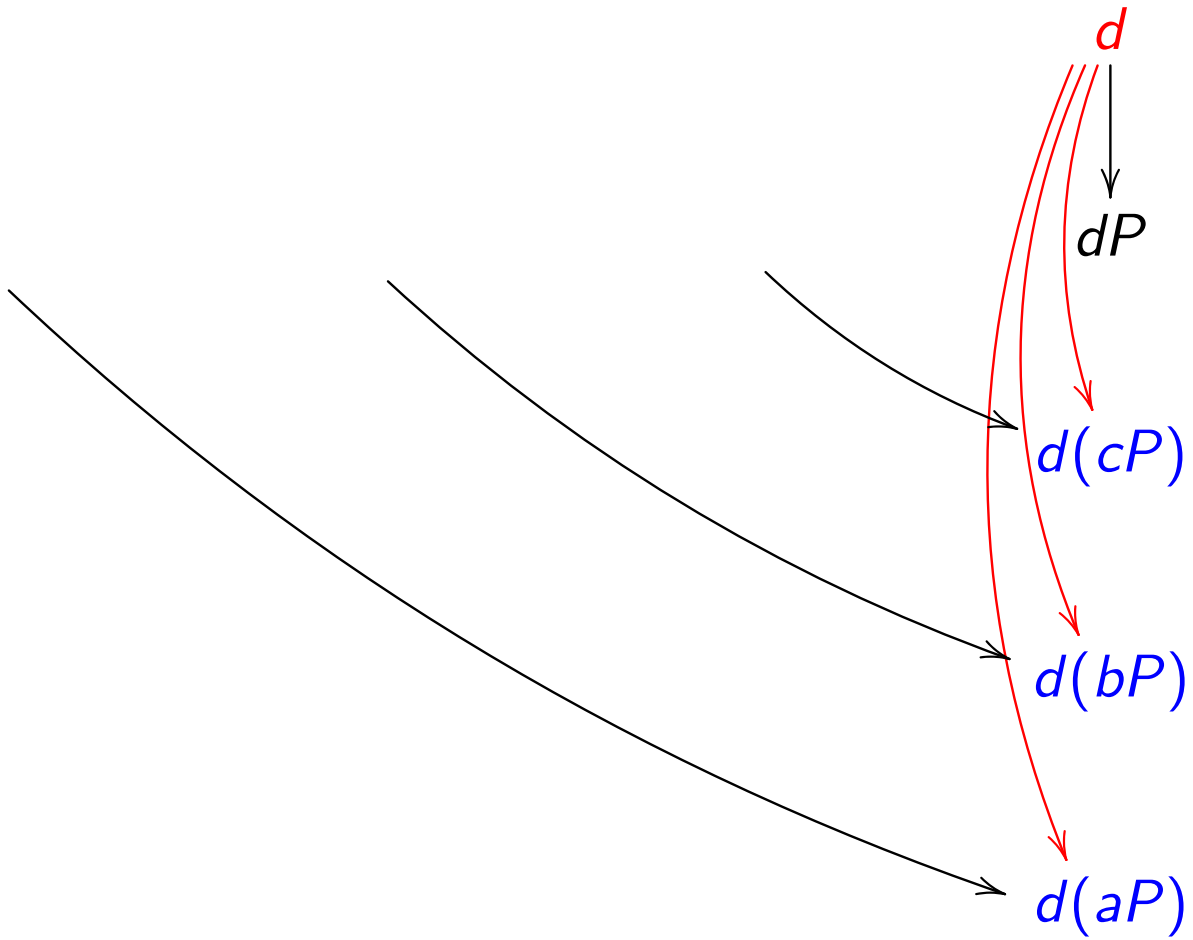


main DH challenge: make variable-base scalar mult as fast as possible

Part I

Kummer strikes back: new DH speed records
ASIACRYPT 2014

Joint work with Daniel J. Bernstein, Tanja Lange, and Peter Schwabe

Ladder

Input: $Q, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R_0 = nQ$

$R_0 \leftarrow 0Q; \quad R_1 \leftarrow 1Q$
if $n_i = 0$ **then**
  $R_0 \leftarrow 2R_0; \quad R_1 \leftarrow R_0 + R_1$
else
  $R_0 \leftarrow R_0 + R_1; \quad R_1 \leftarrow 2R_1$

Return R_0

Example: Compute $9Q$

$9 = 1001_2$

$(R_0, R_1) = (0Q, 1Q)$

$n_3 = 1 : \quad (1Q, 2Q)$

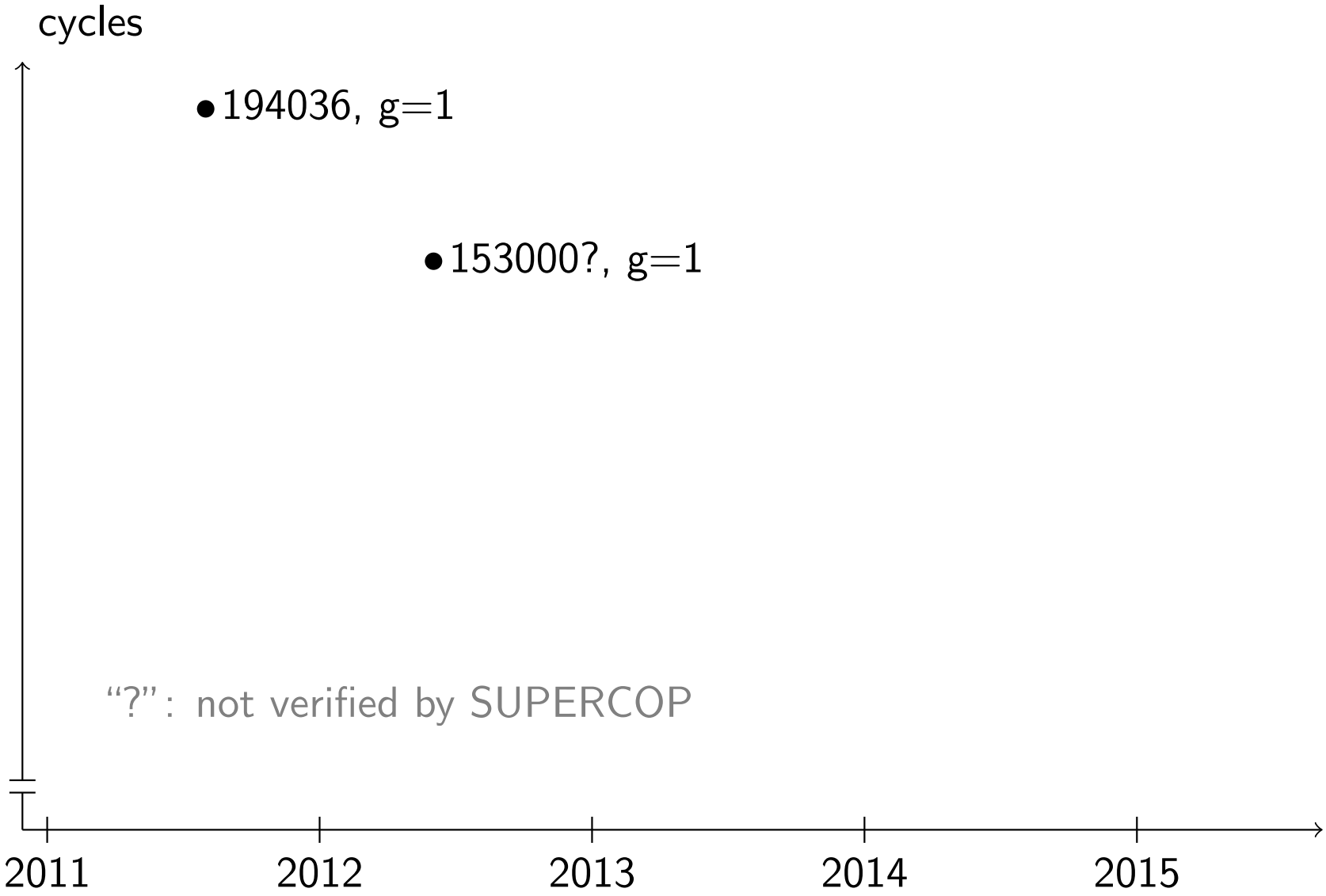
$n_2 = 0 : \quad (2Q, 3Q)$

$n_1 = 0 : \quad (4Q, 5Q)$

$n_0 = 1 : \quad (9Q, 10Q)$

Elliptic VS Hyperelliptic

Sandy Bridge at high security level and (claimed) constant time



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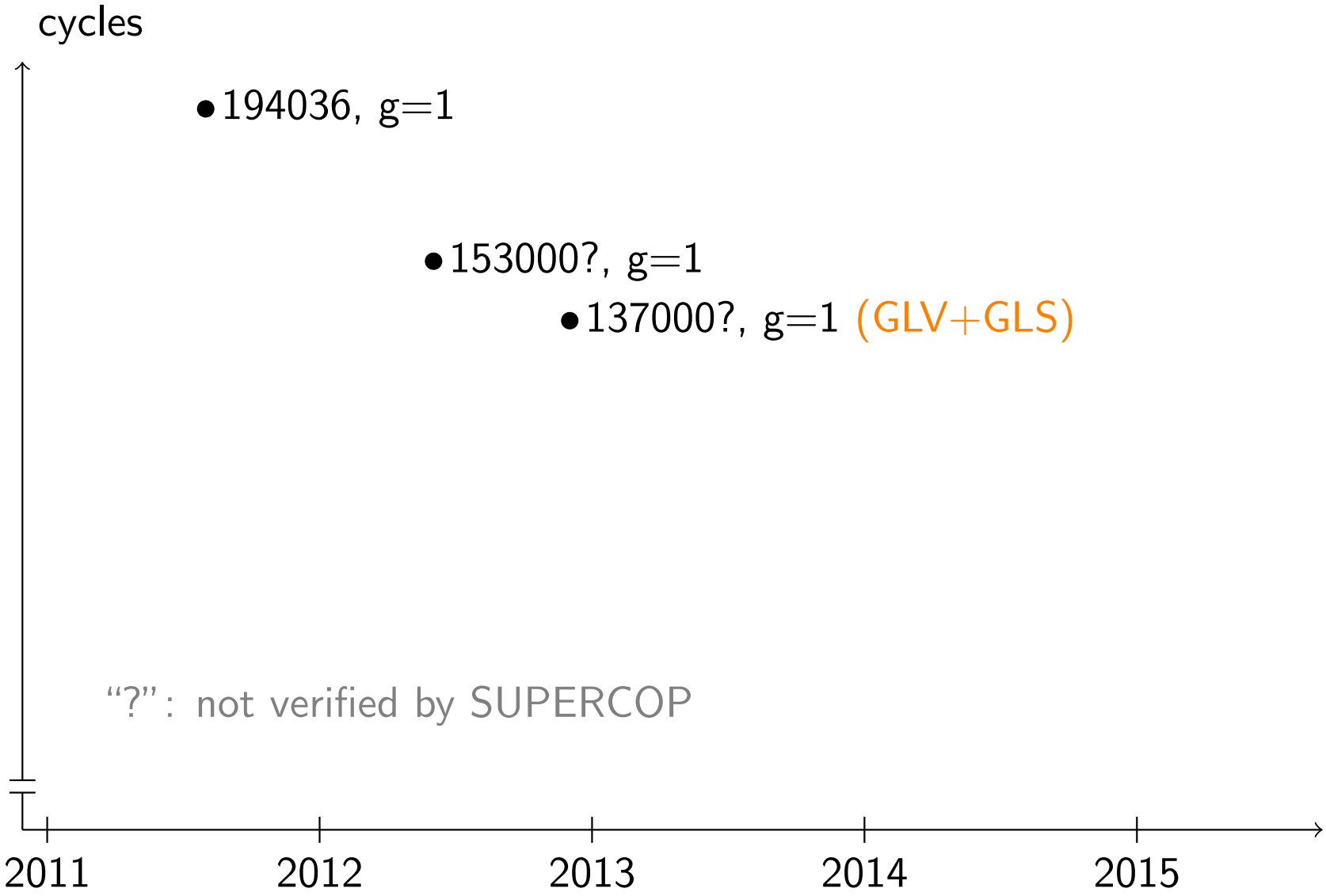

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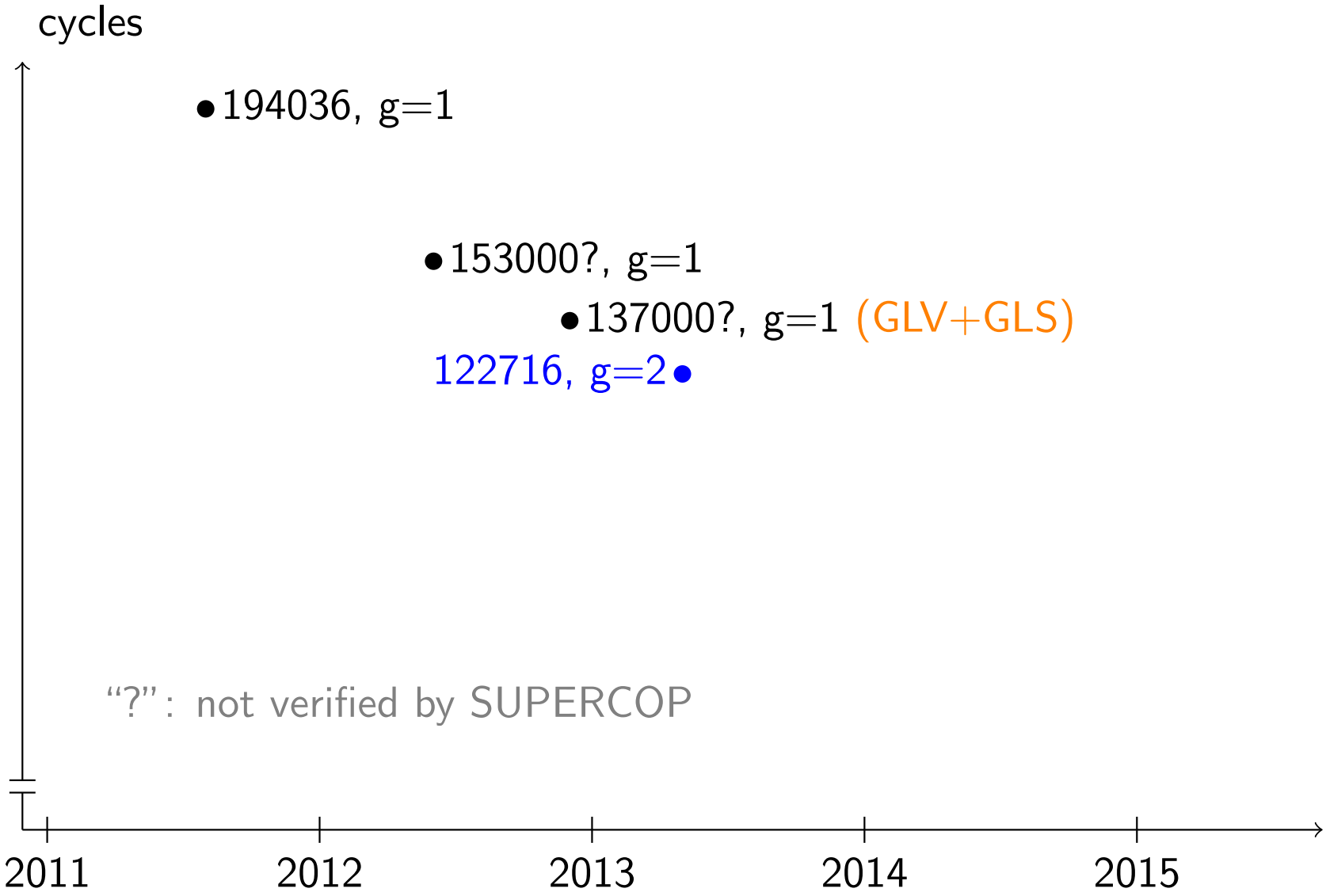

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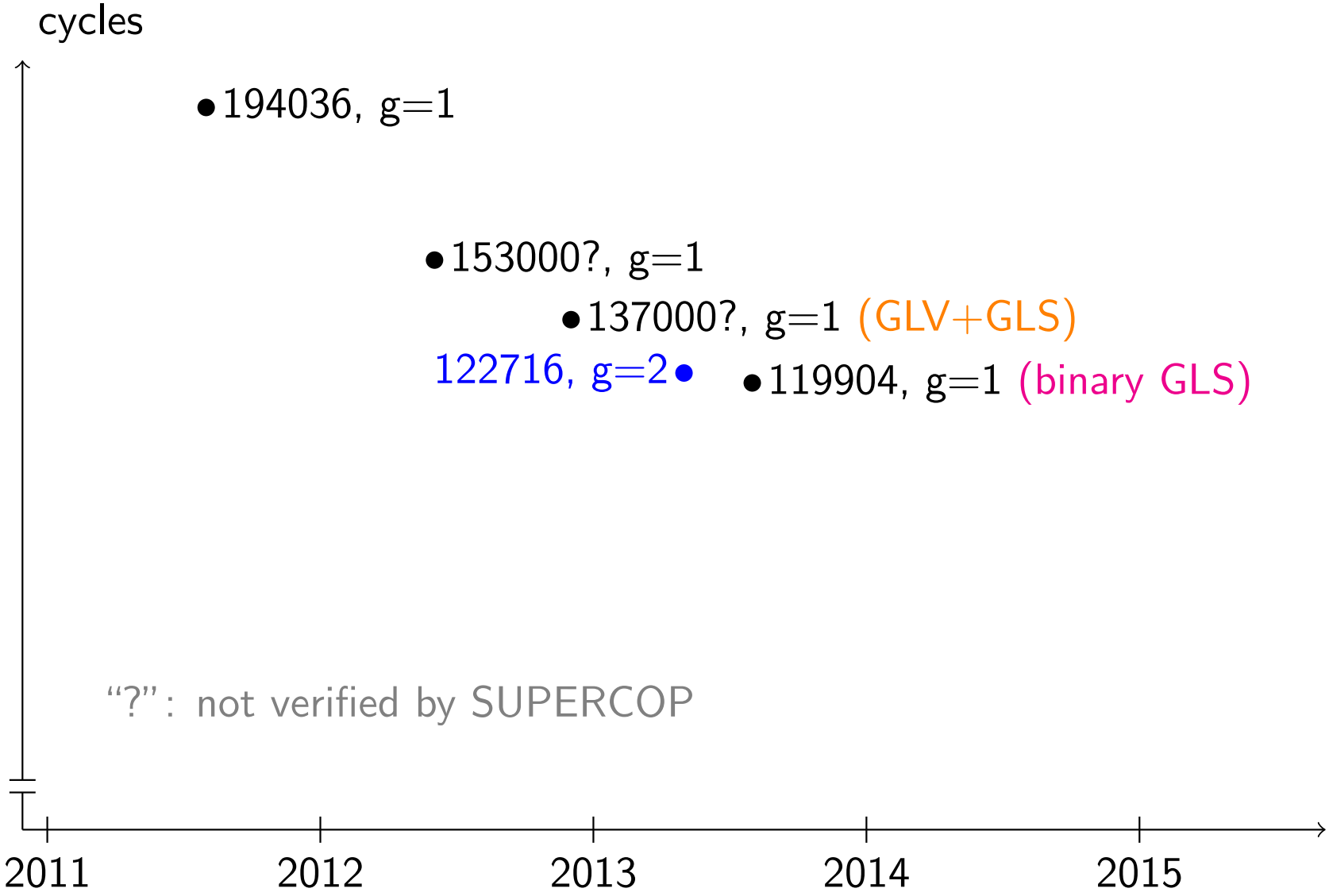

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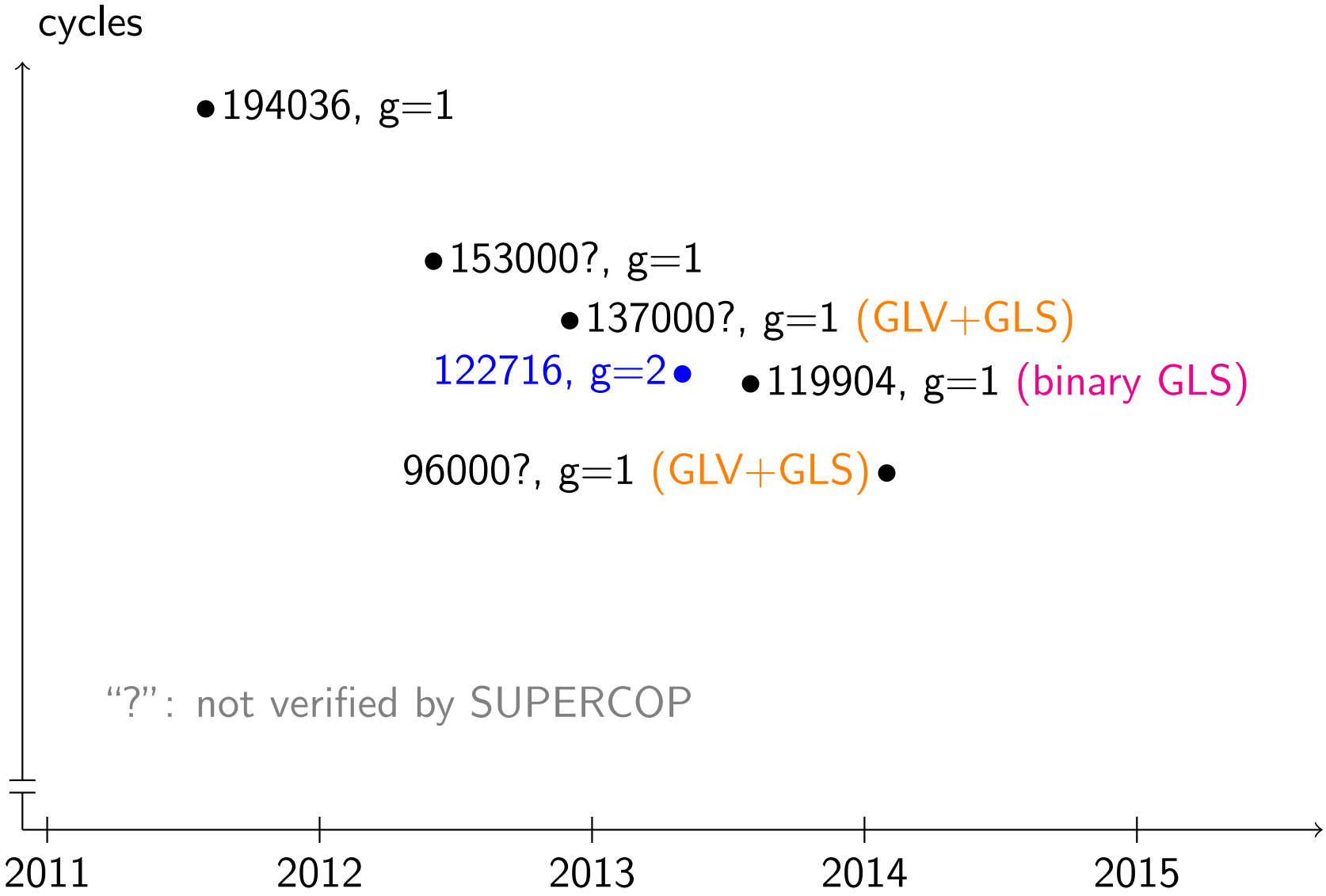

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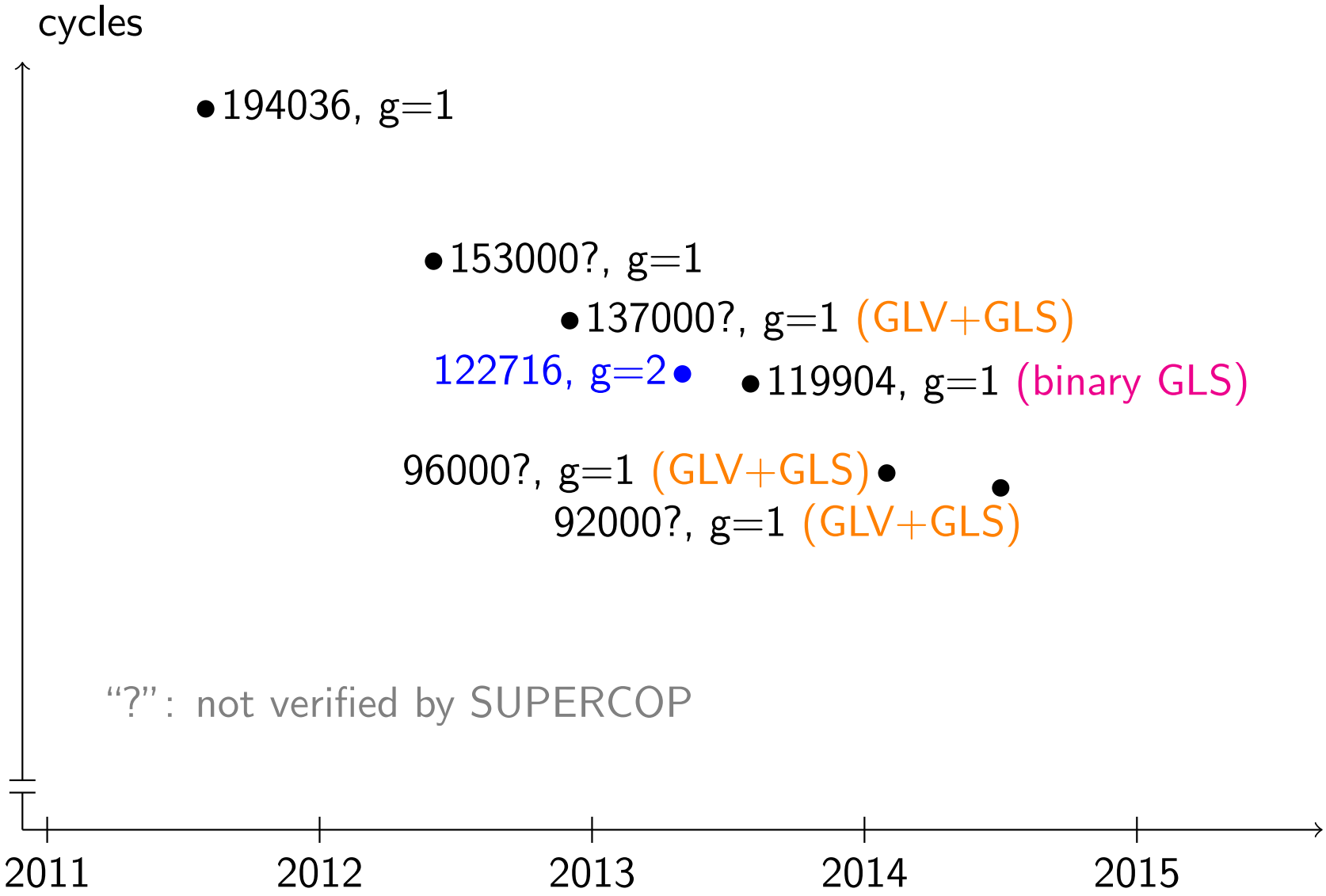

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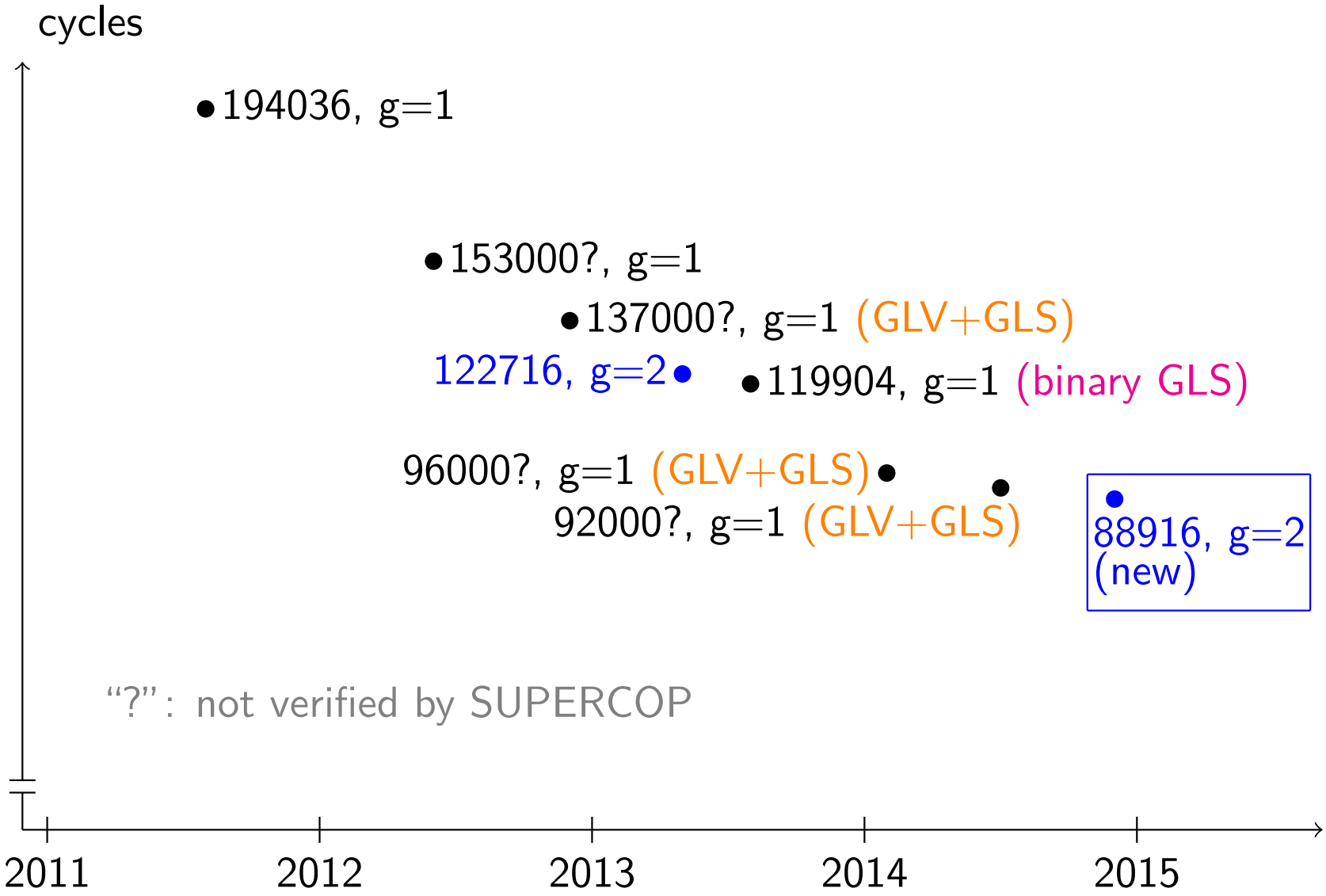
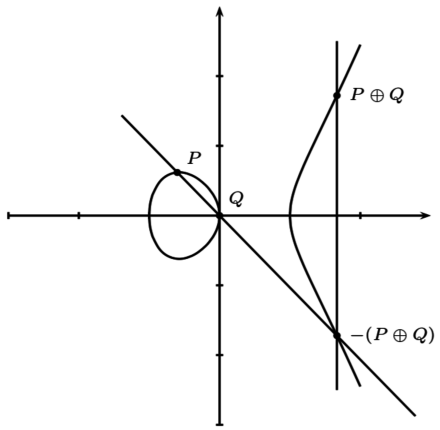


Elliptic-Hyperelliptic Analogy

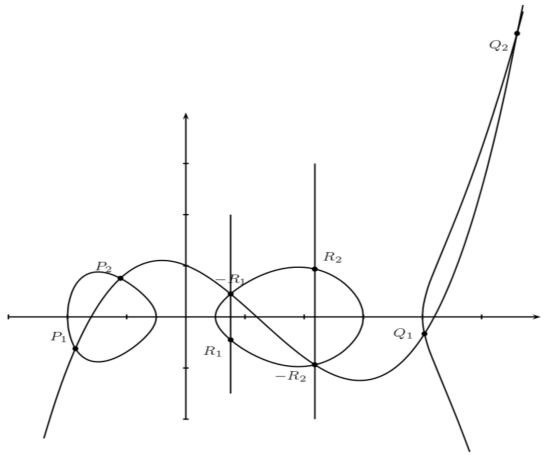
ECC



$y^2 = x^3 + ax + b$

$\longrightarrow$ x-line represented as $(X : Z)$

HECC



$v^2 = u^5 + f_4u^4 + f_3u^3 + f_2u^2 + f_1u^1 + f_0$

$\longrightarrow$ Kummer surface represented as $(X : Y : Z : T)$

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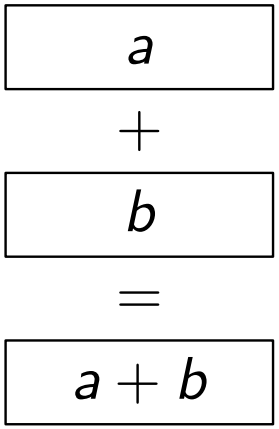
  - 6 of 25 multiplications are by *small* constant
- new: formulas well suited for vectorization

Vectorization

without vector



Vectorization

without vector

a
$+$
b
$=$
$a + b$

with vector

a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$
$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$	$a_3 + b_3$

Vectorization

without vector

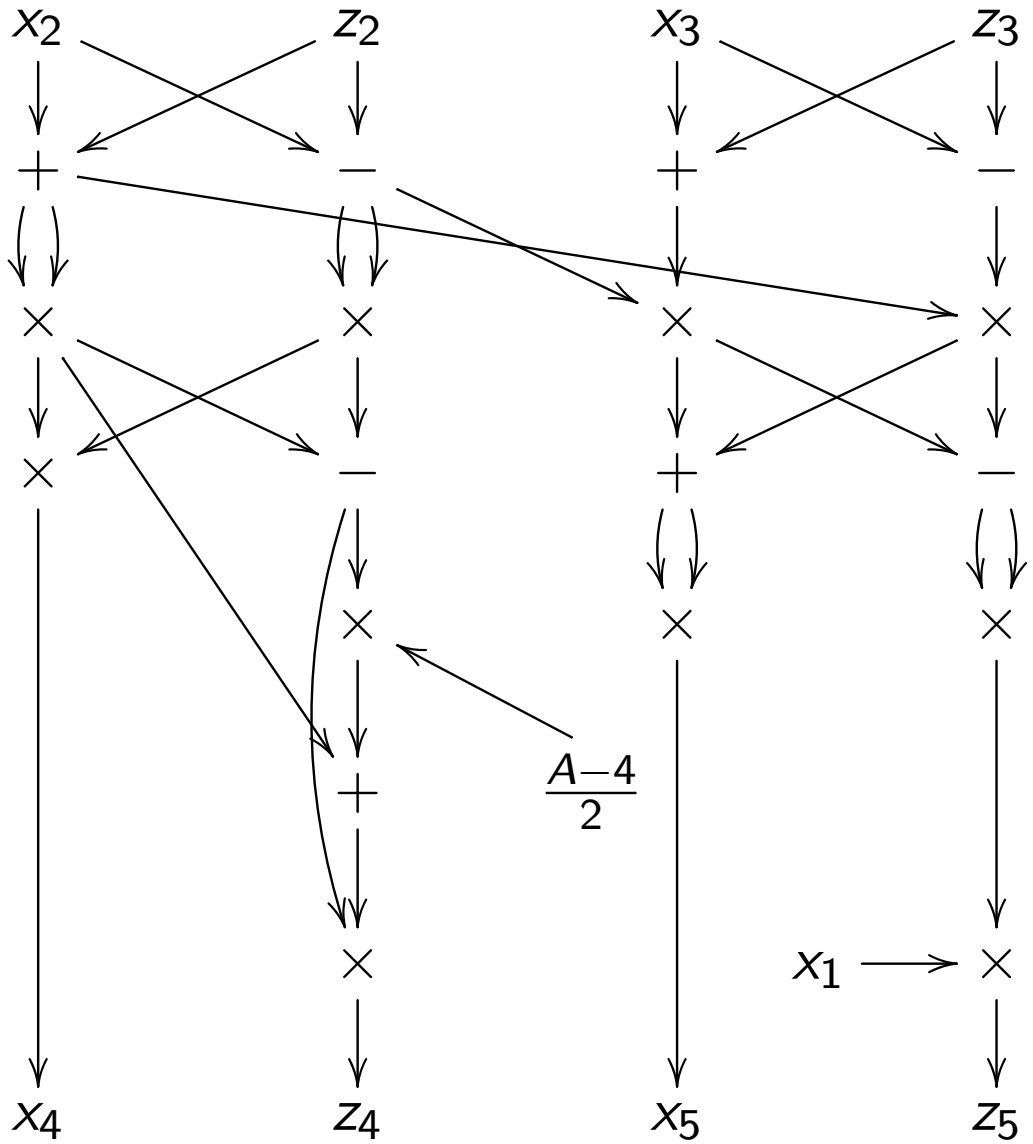
a
$+$
b
$=$
$a + b$

with vector

a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$
$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$	$a_3 + b_3$

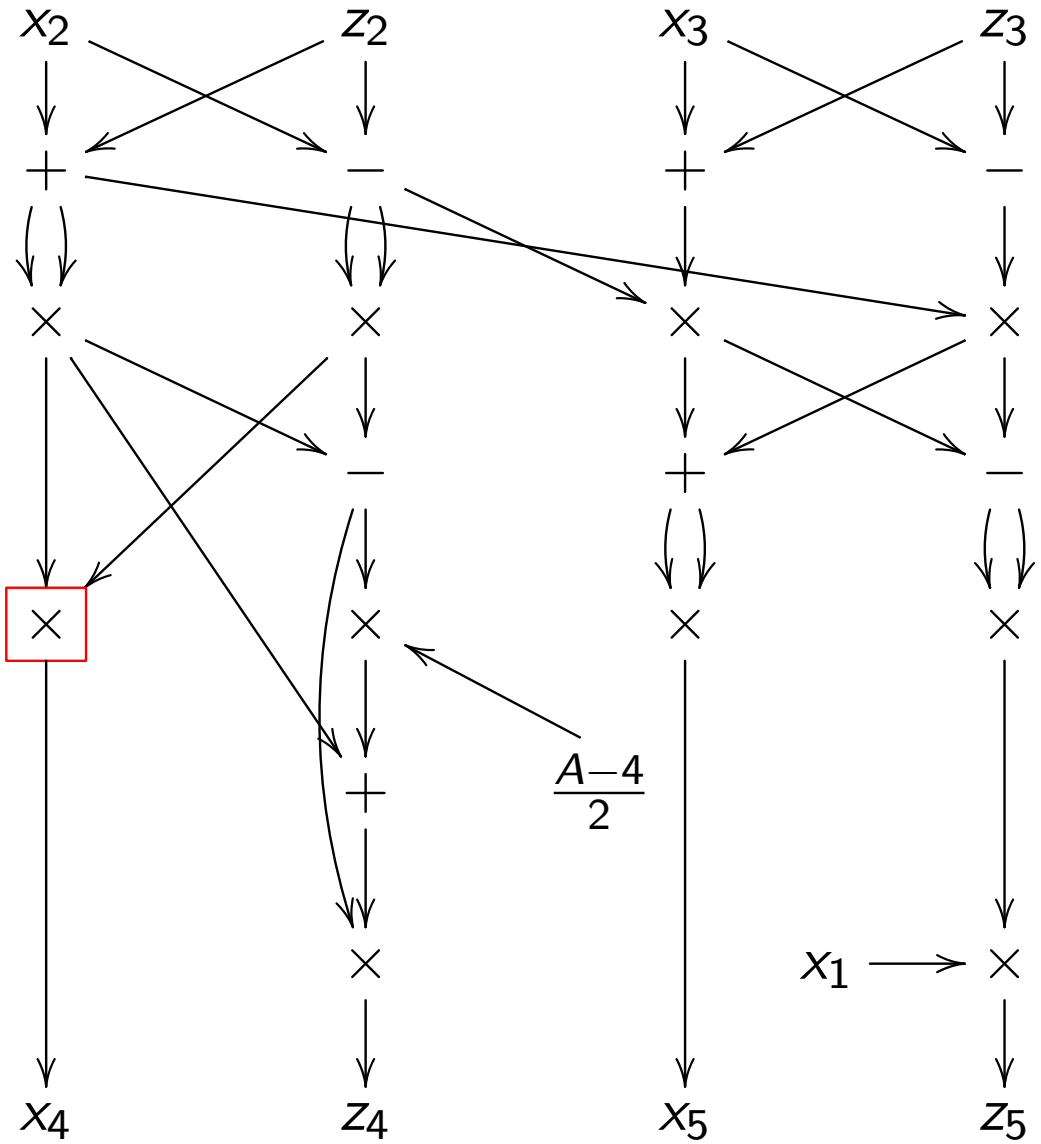
- **single** instruction performing n **independent** operations on **aligned** inputs

ECC Montgomery Ladder (original)



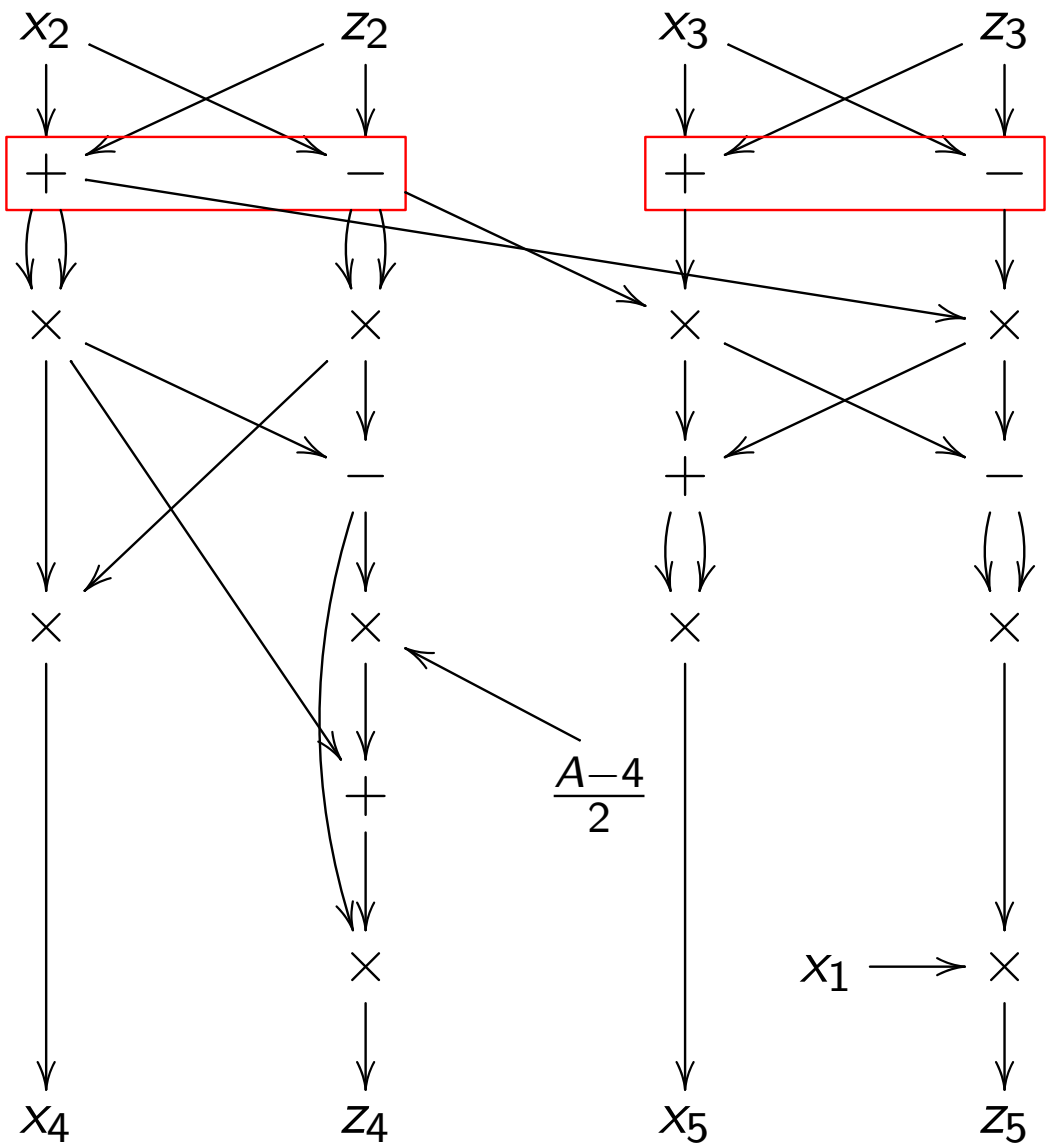
ECC Montgomery Ladder (2-way vectorization)

- move $\times$ down



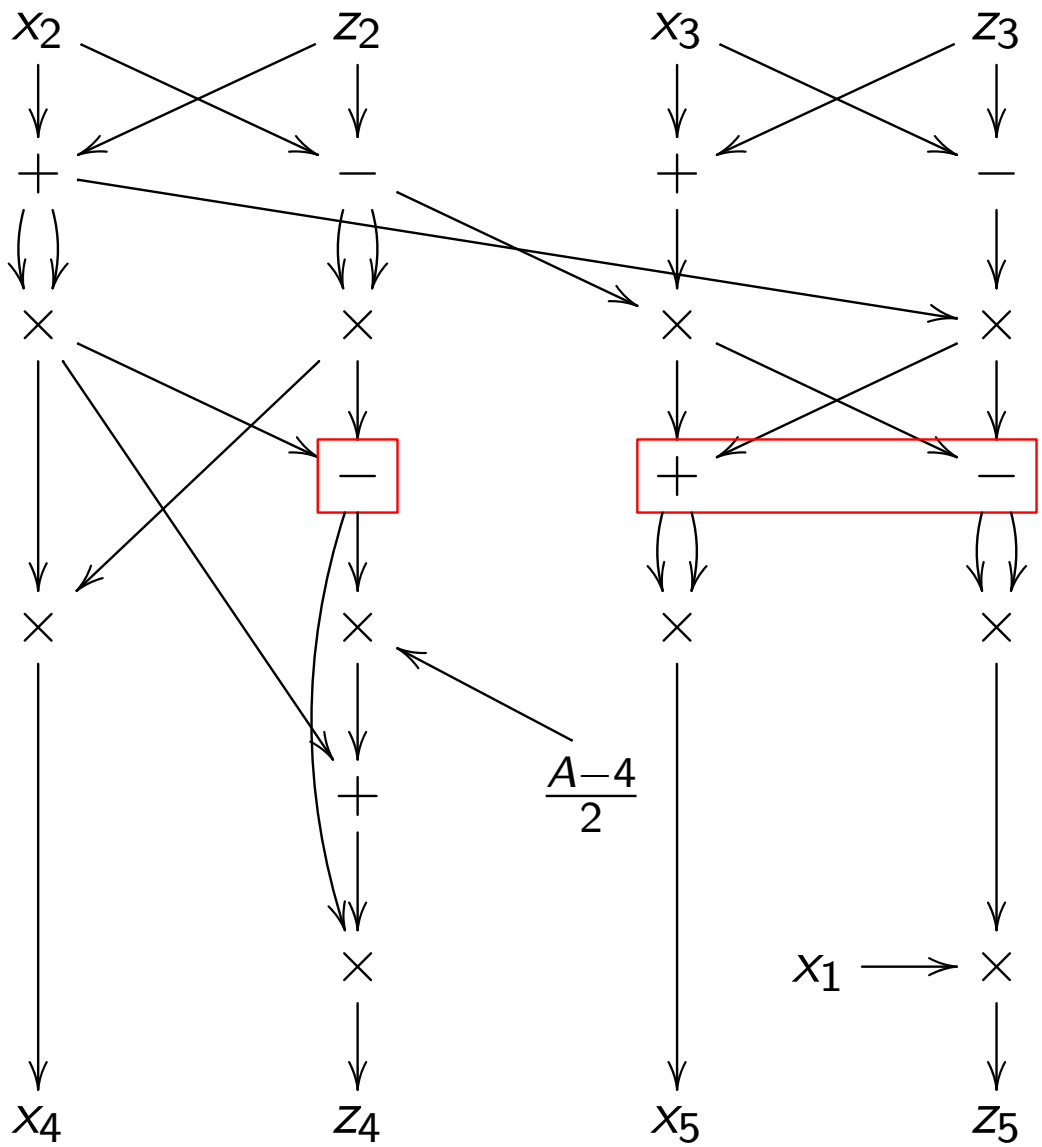
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move $\times$ down



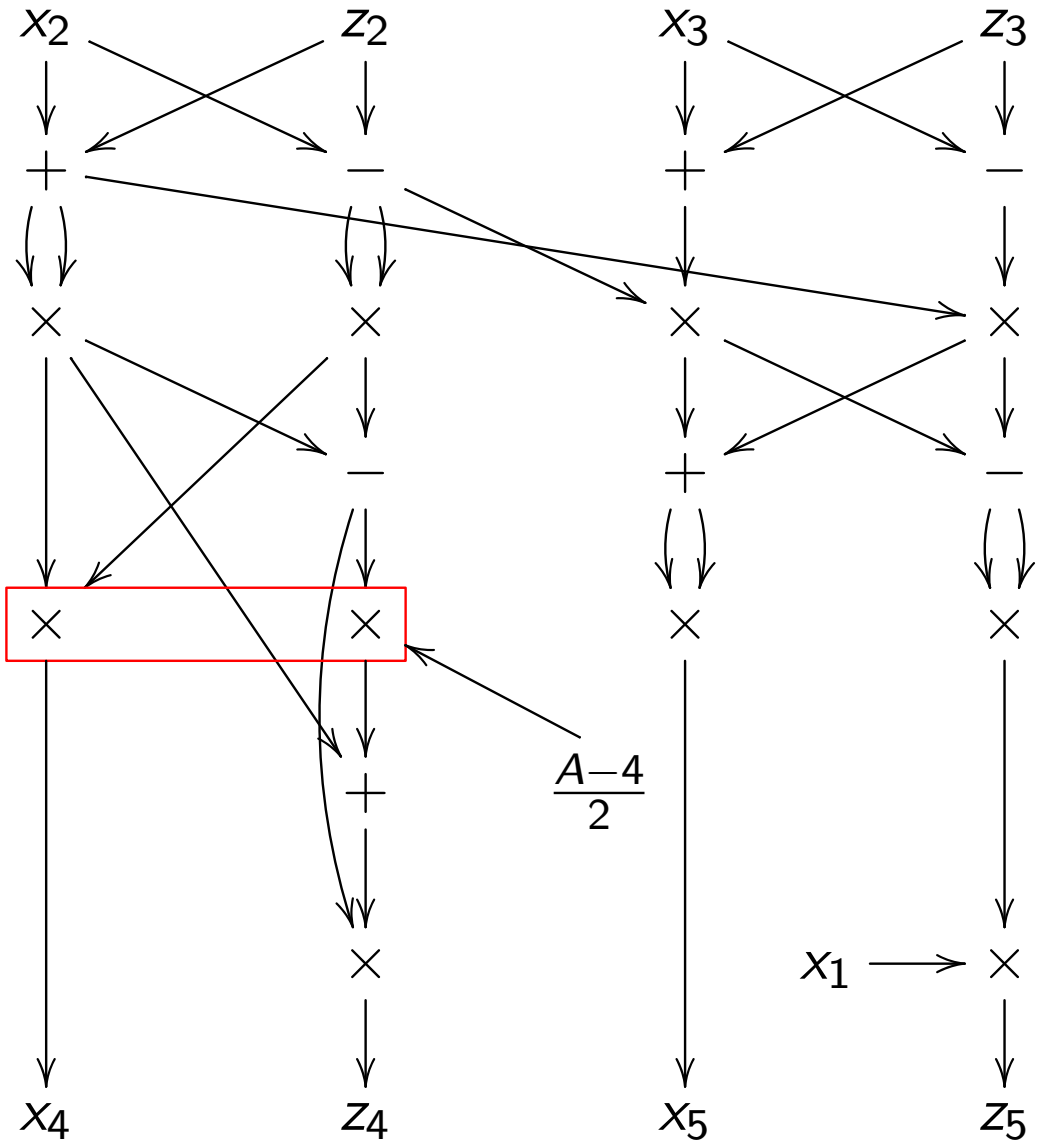
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move $\times$ down
- require permutation and leave $+$ idle



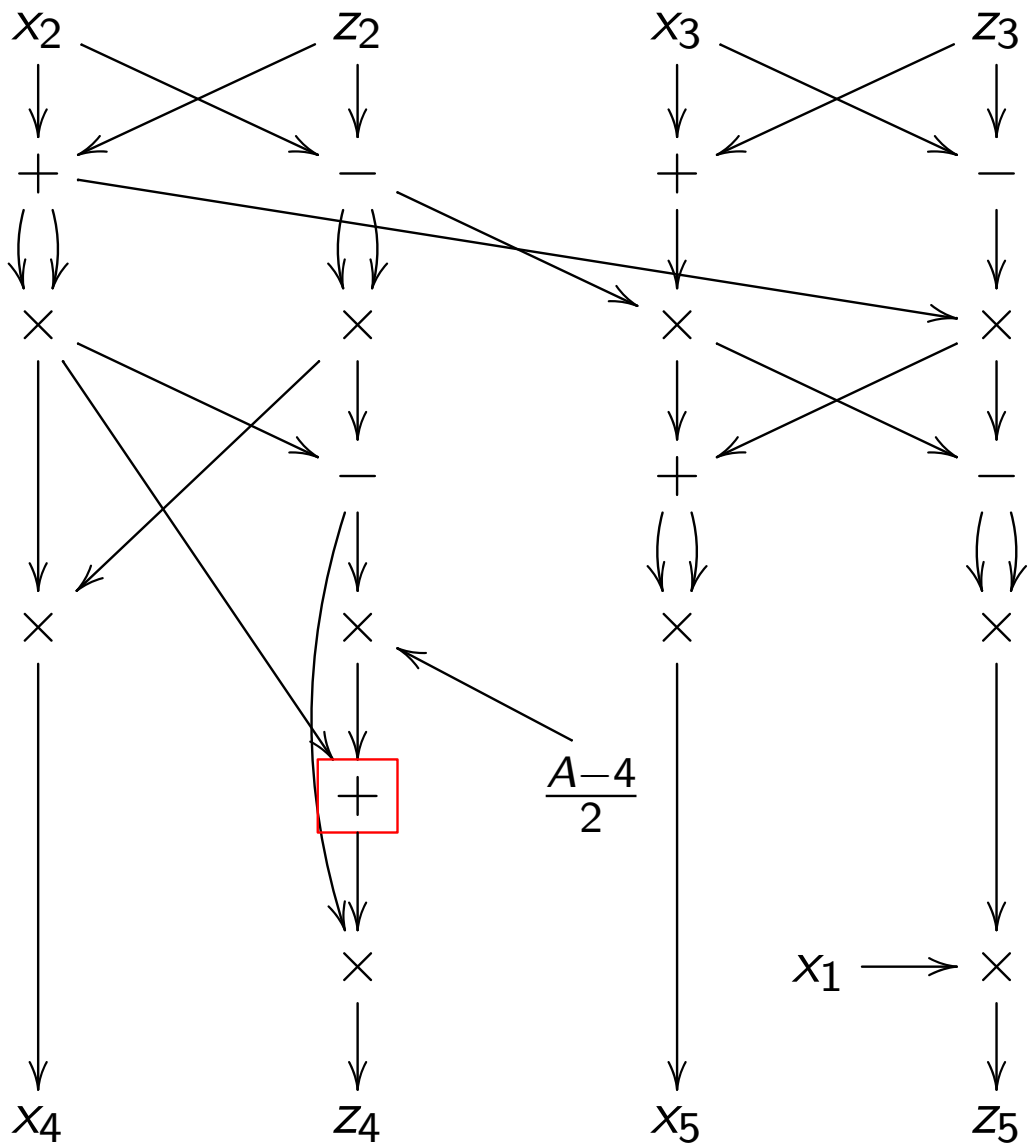
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move $\times$ down
- require permutation and leave $+$ idle
- slow down mult. by constant



ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move $\times$ down
- require permutation and leave $+$ idle
- slow down mult. by constant
- nothing to match $+$



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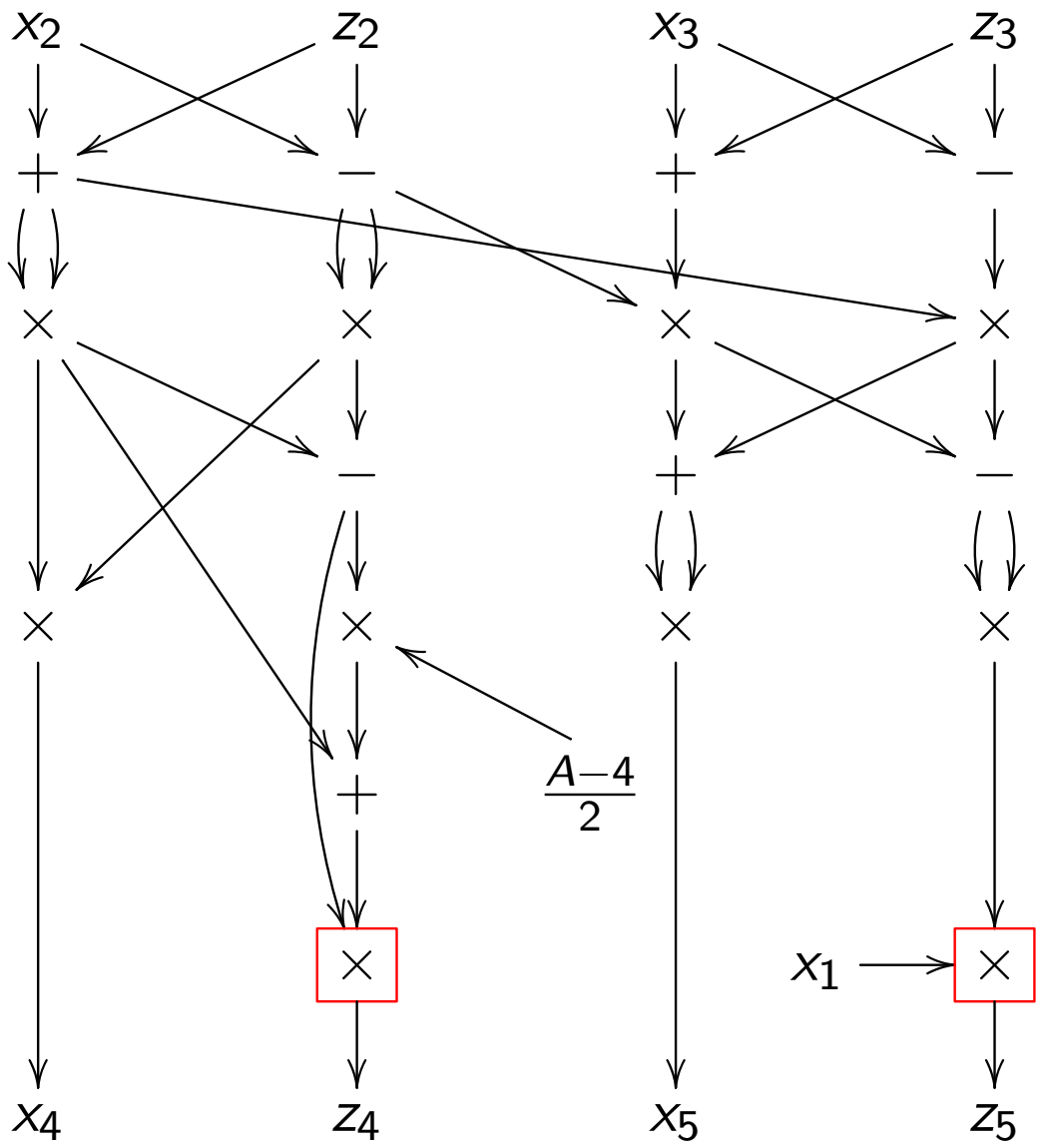
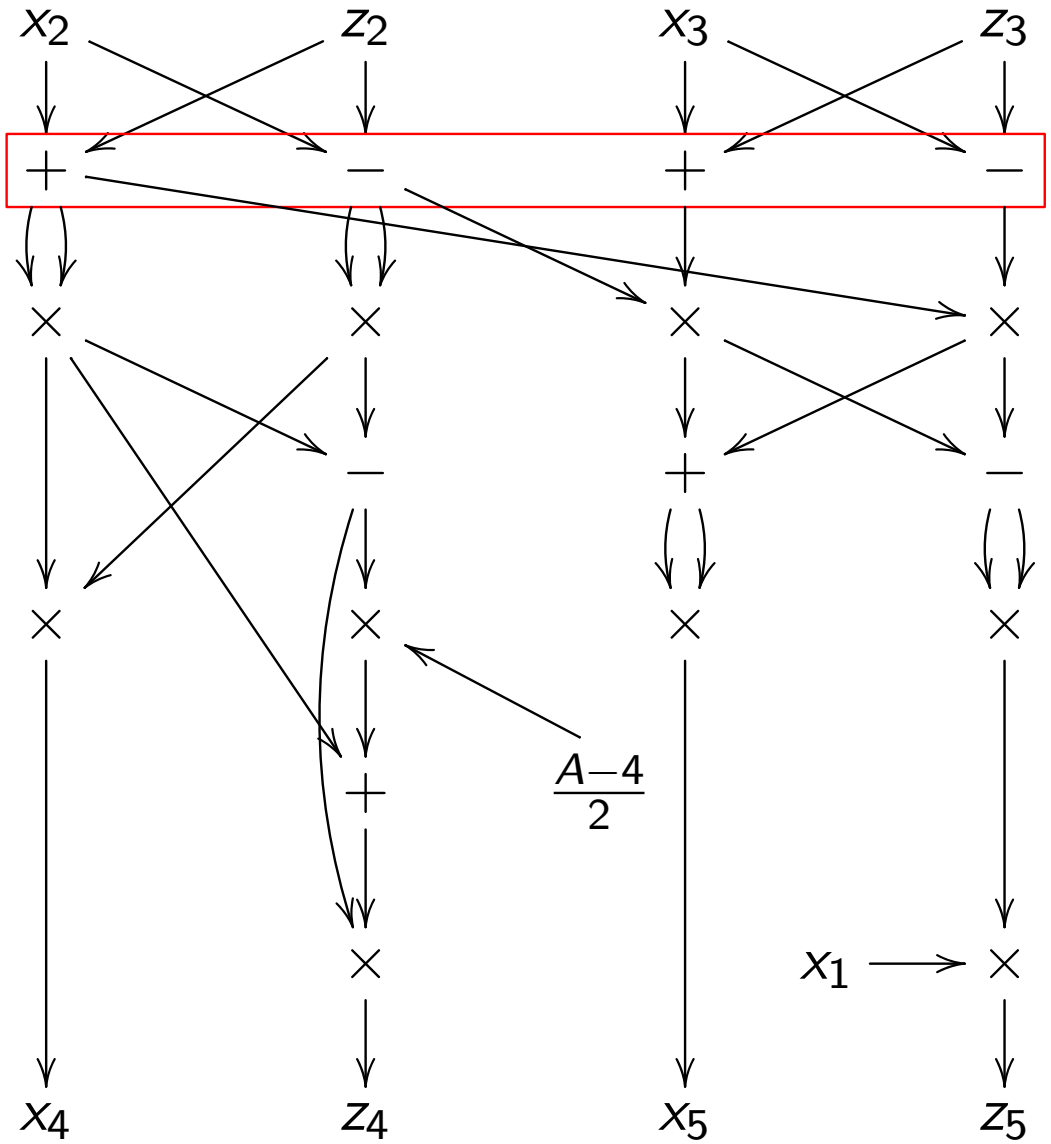
- nothing to match $+$
- require permutation



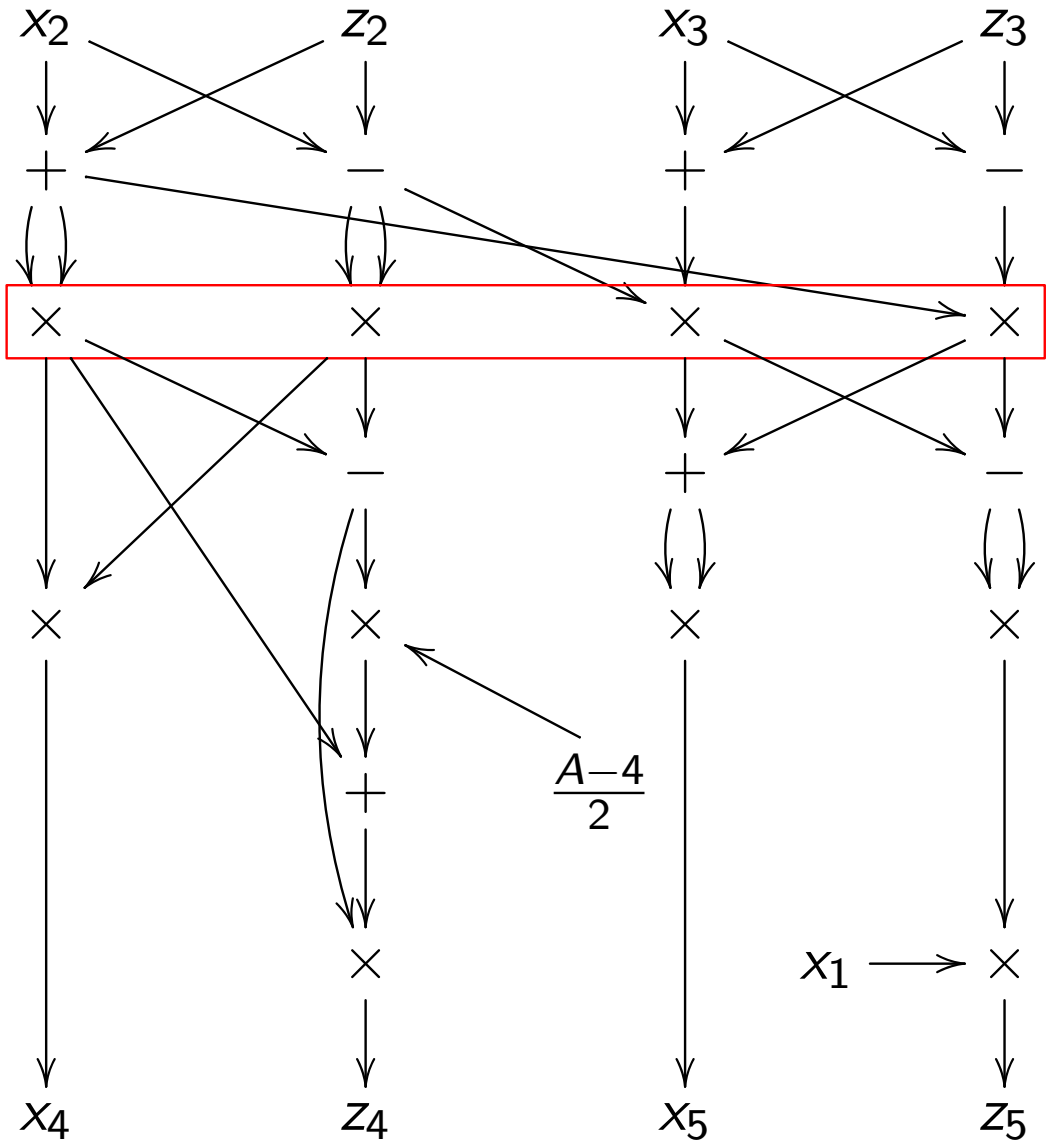
ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$



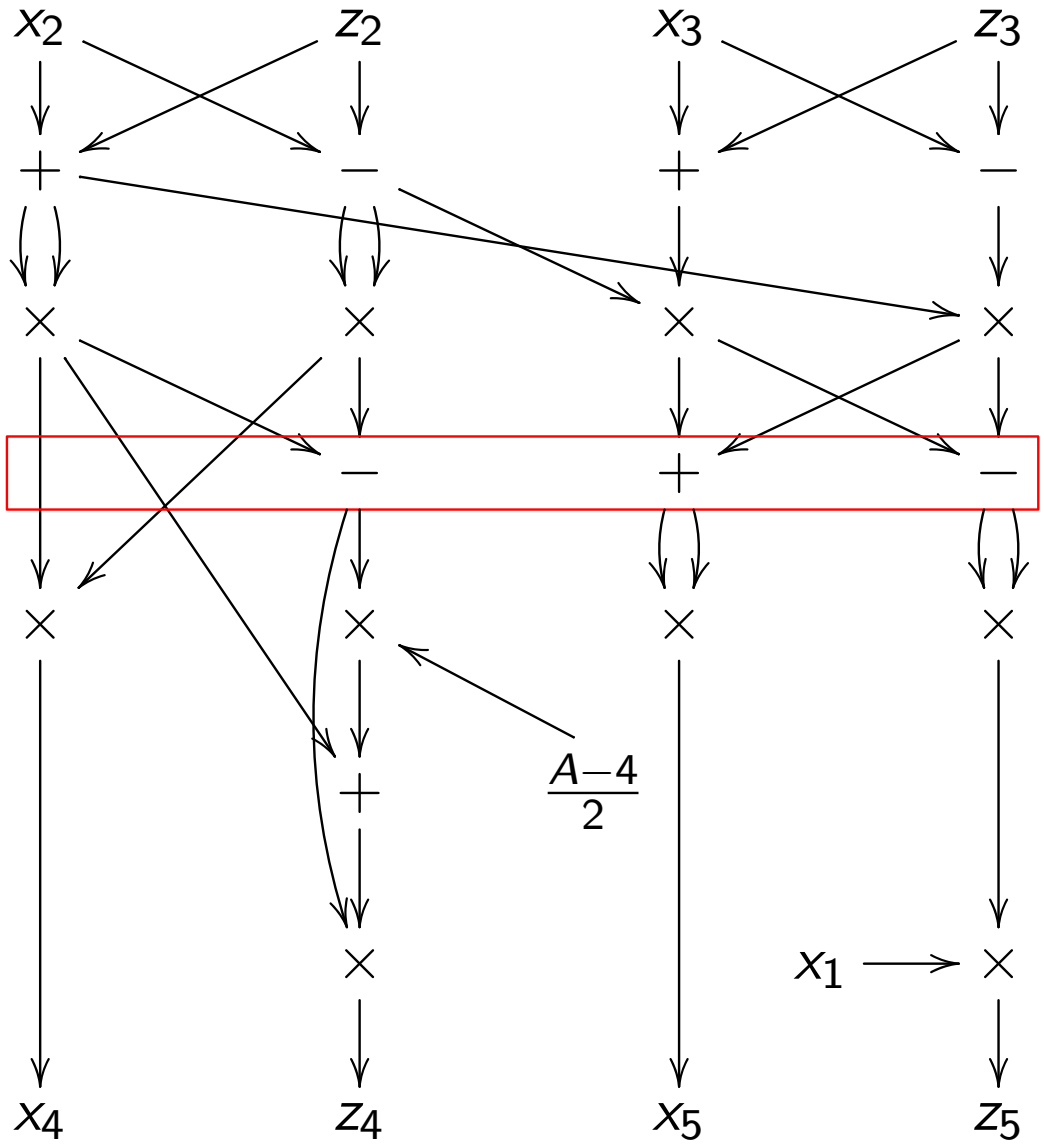
ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
- slow down squaring



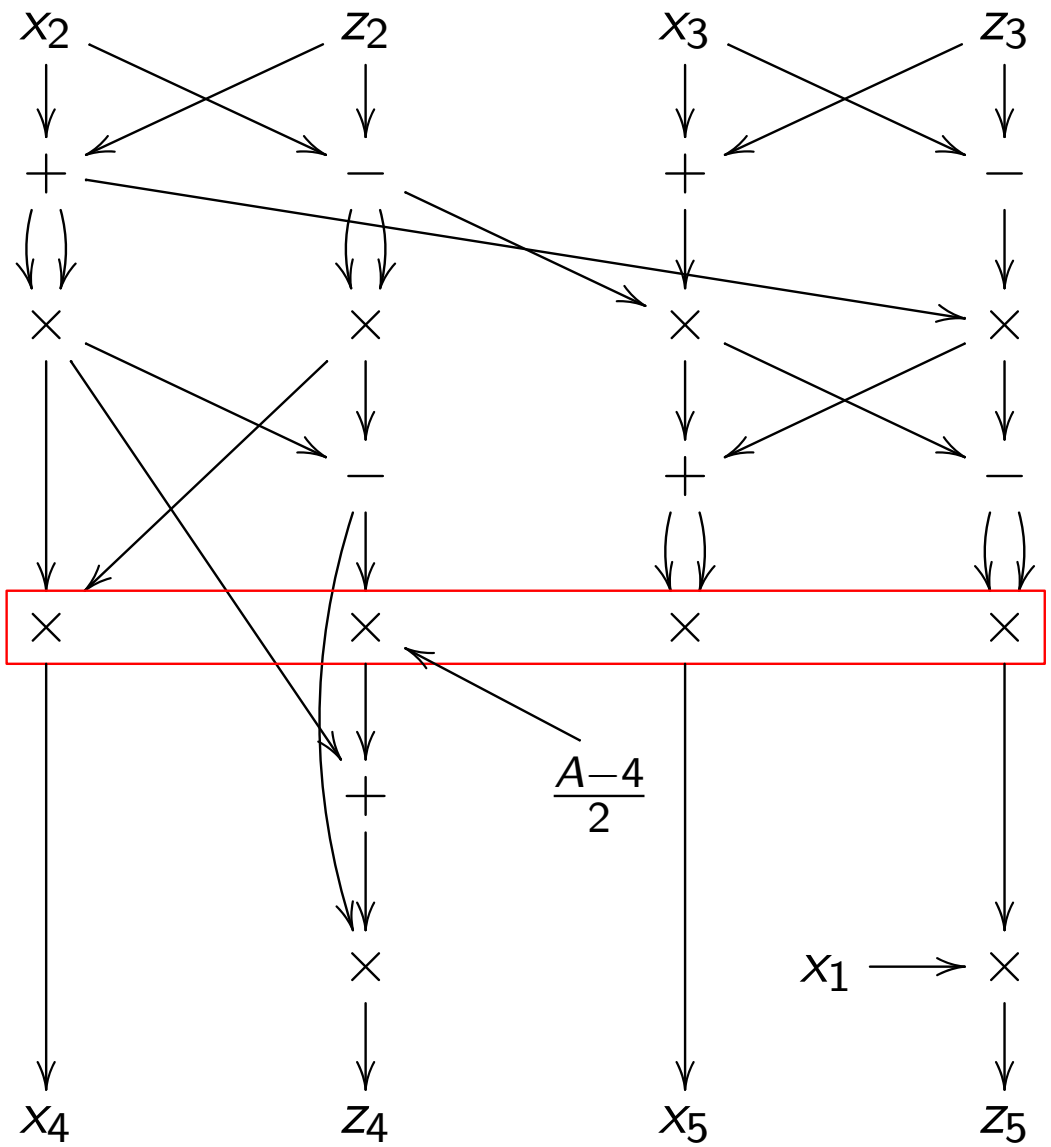
ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
- slow down squaring
- nothing to match and match $+$ and $-$



ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
- slow down squaring
- nothing to match and match $+$ and $-$
- slow down mult. by constant and squaring



ECC Montgomery Ladder (4-way vectorization)

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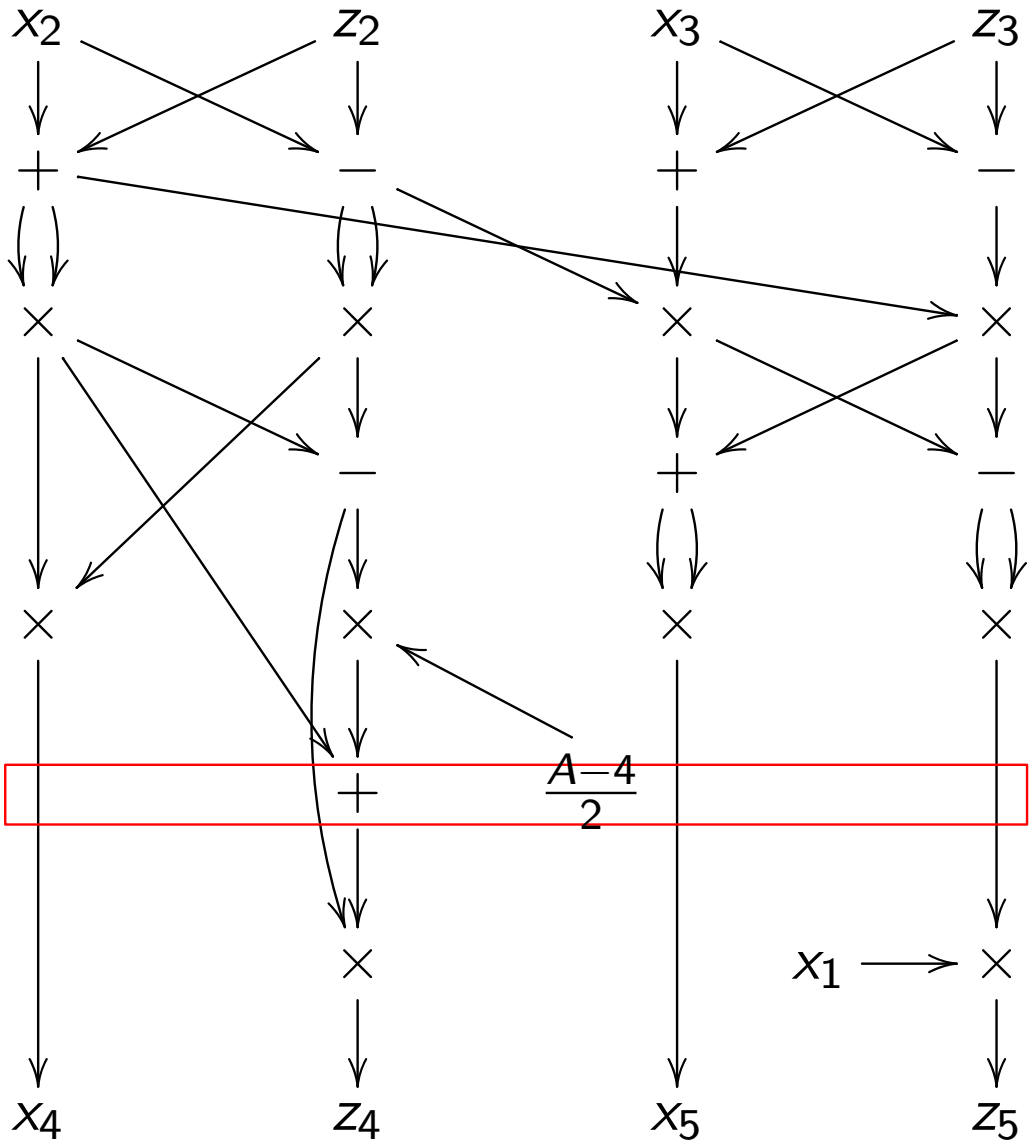
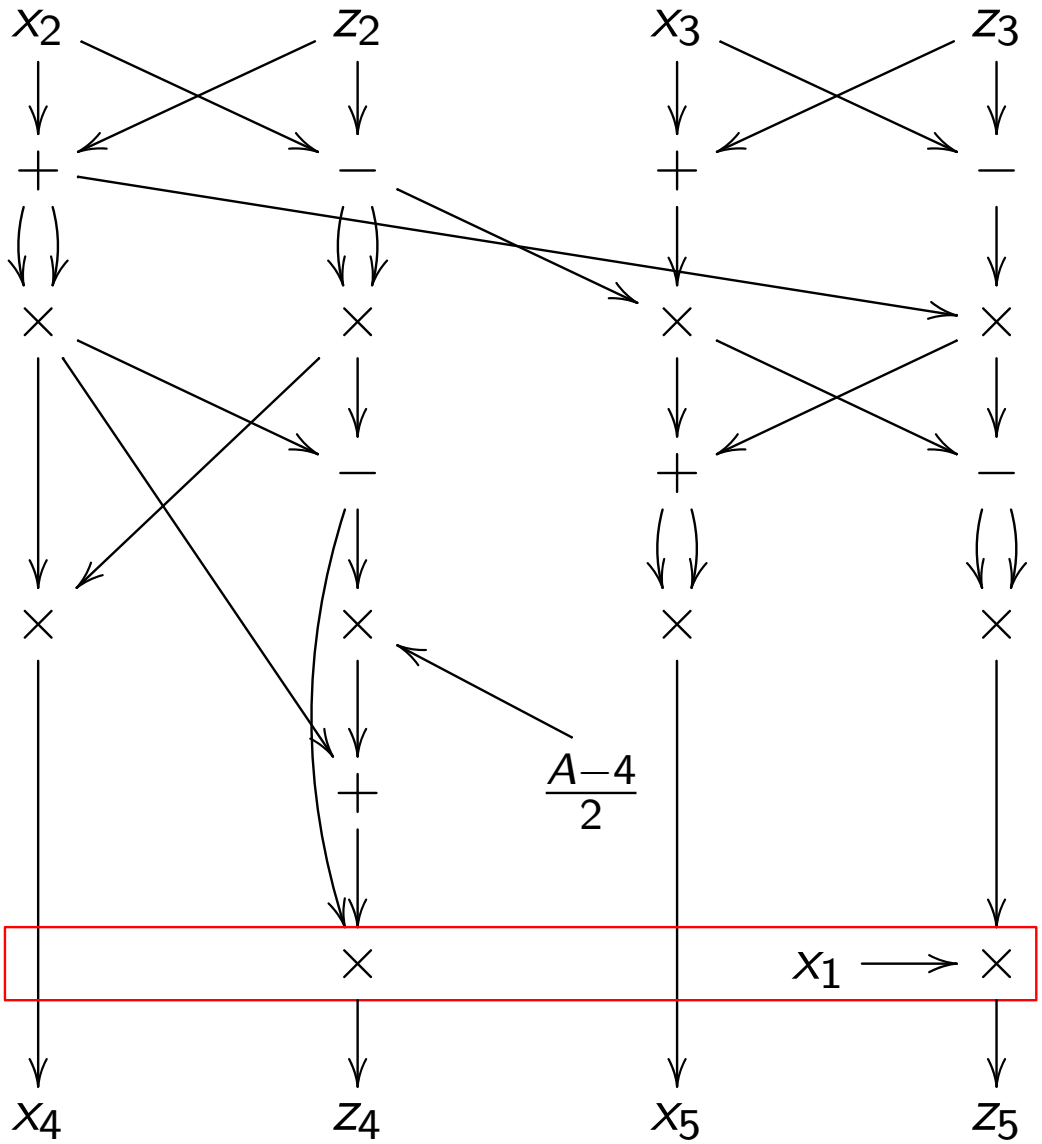
- nothing to match and match $+$ and $-$
- slow down mult. by constant and squaring
- nothing to match $+$

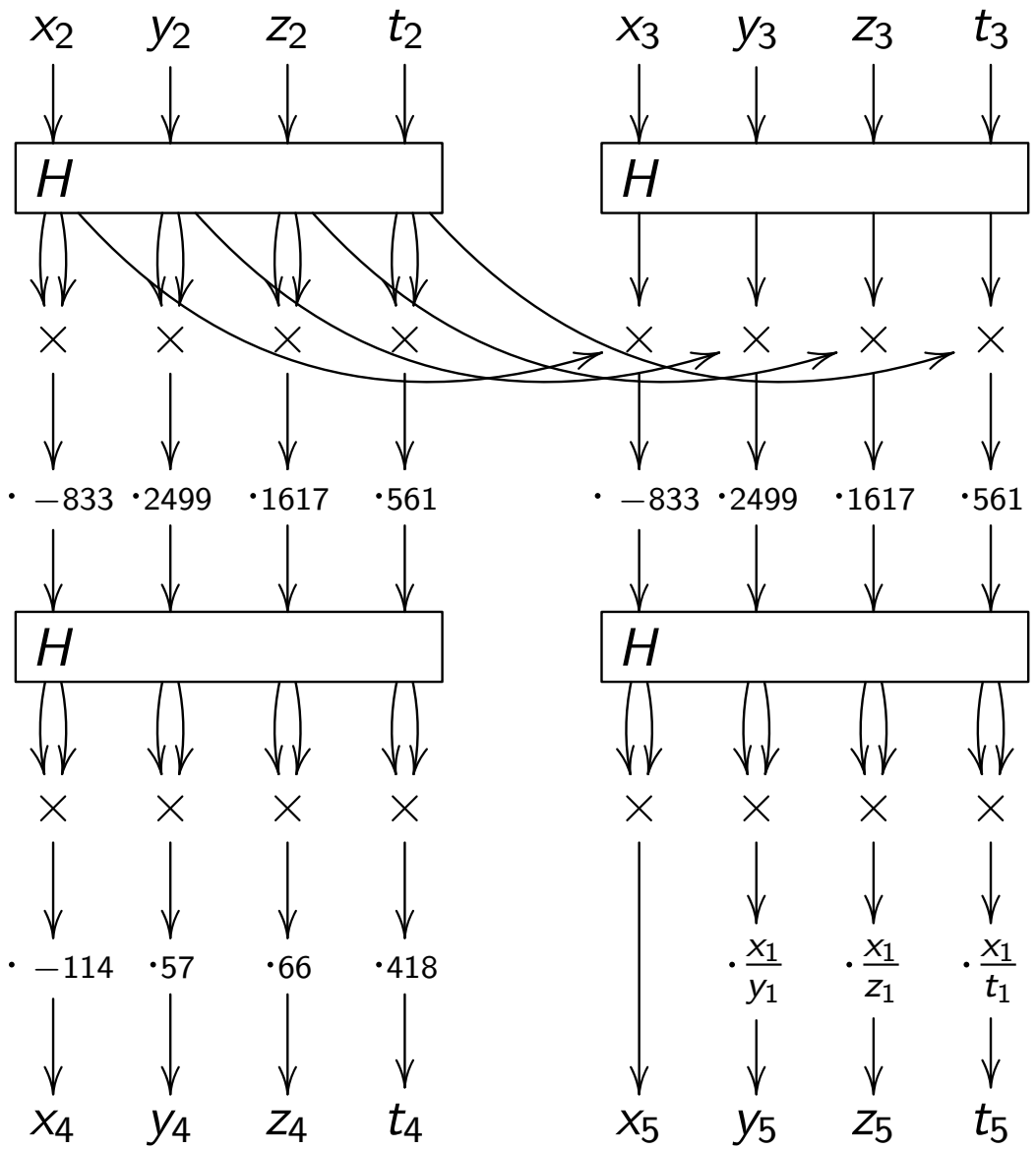


ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
- slow down squaring
- nothing to match and match $+$ and $-$
- slow down mult. by constant and squaring
- nothing to match $+$
- nothing to match $\times$



Squared Kummer Surface Ladder



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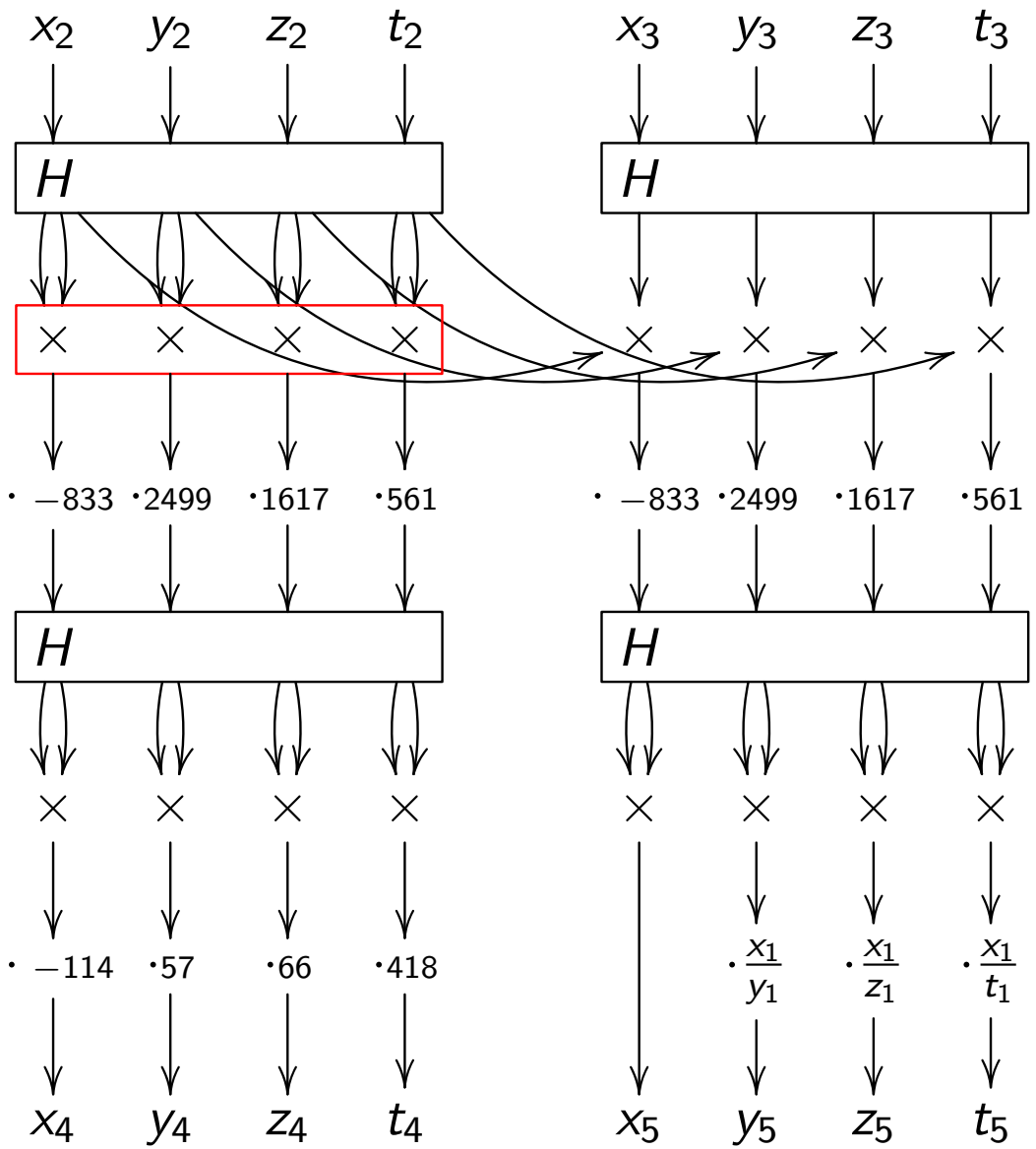

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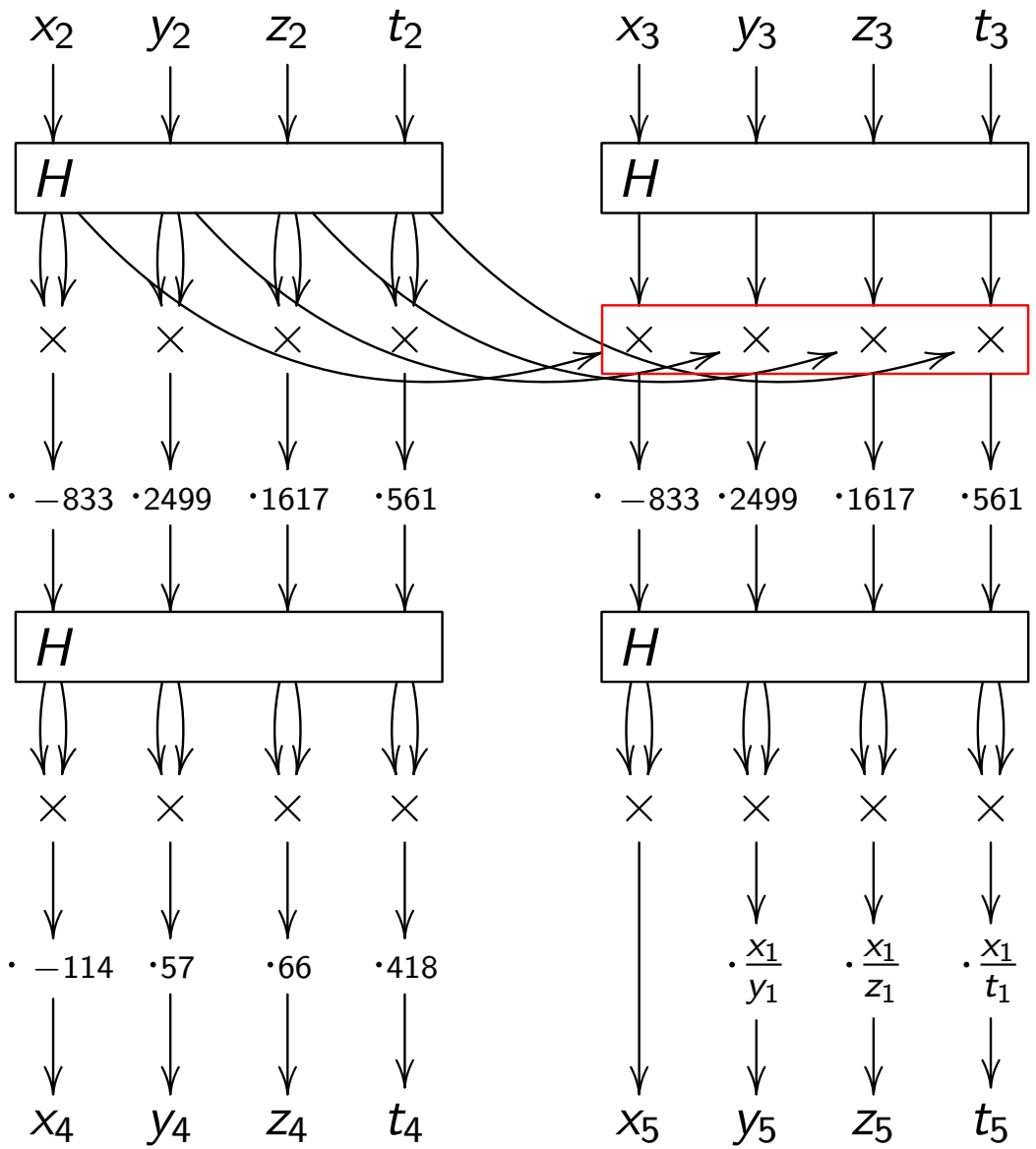

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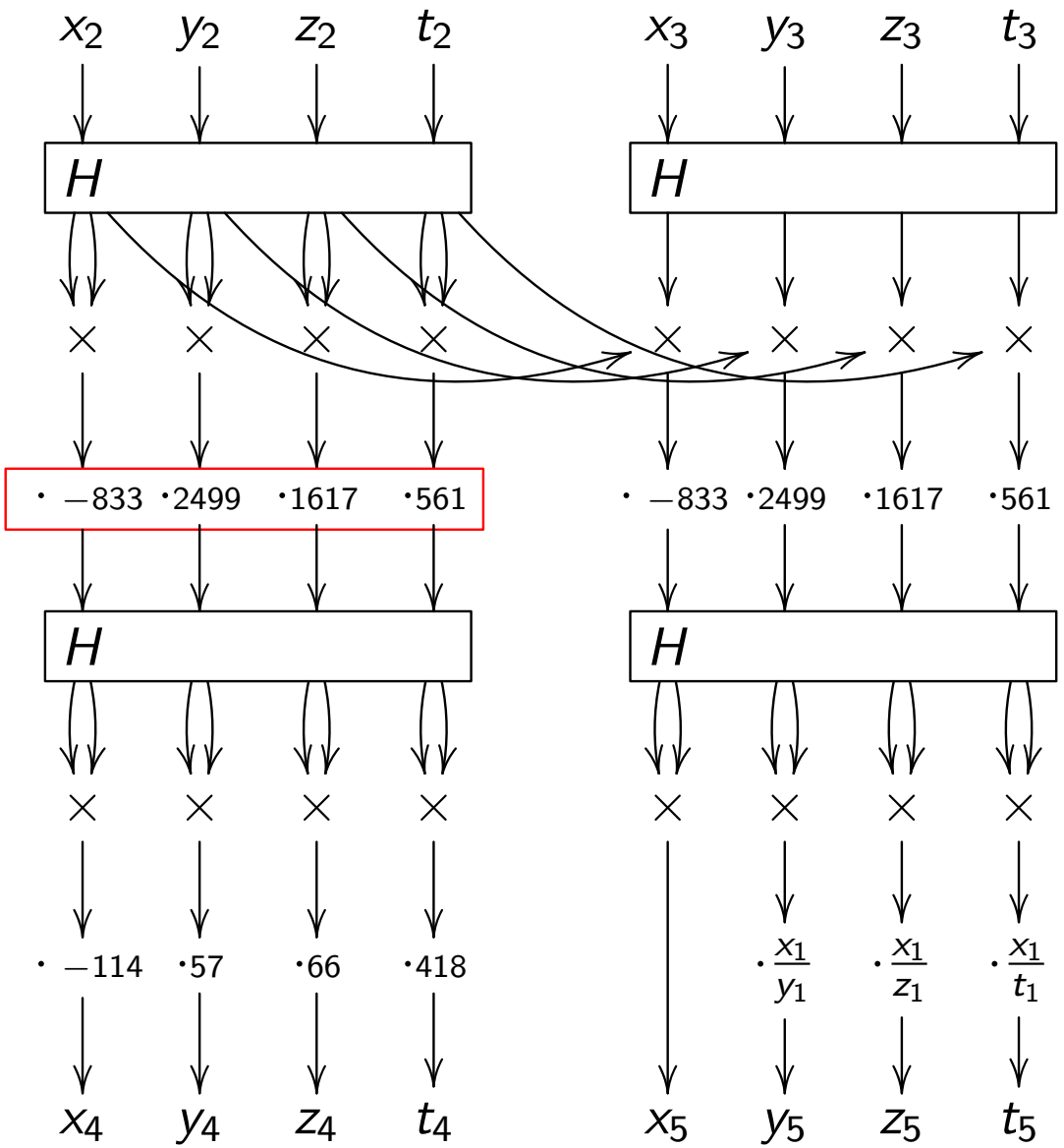

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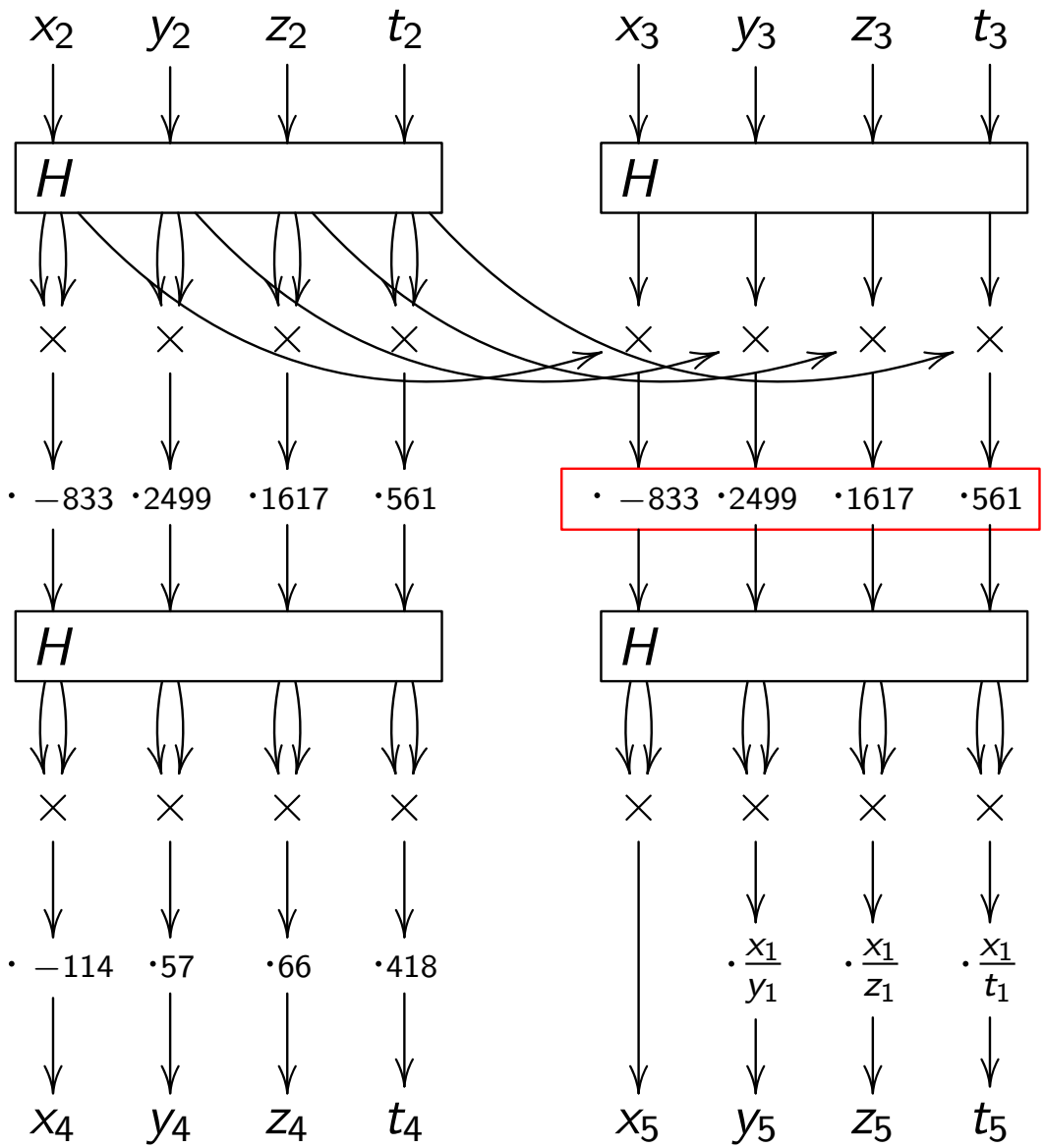

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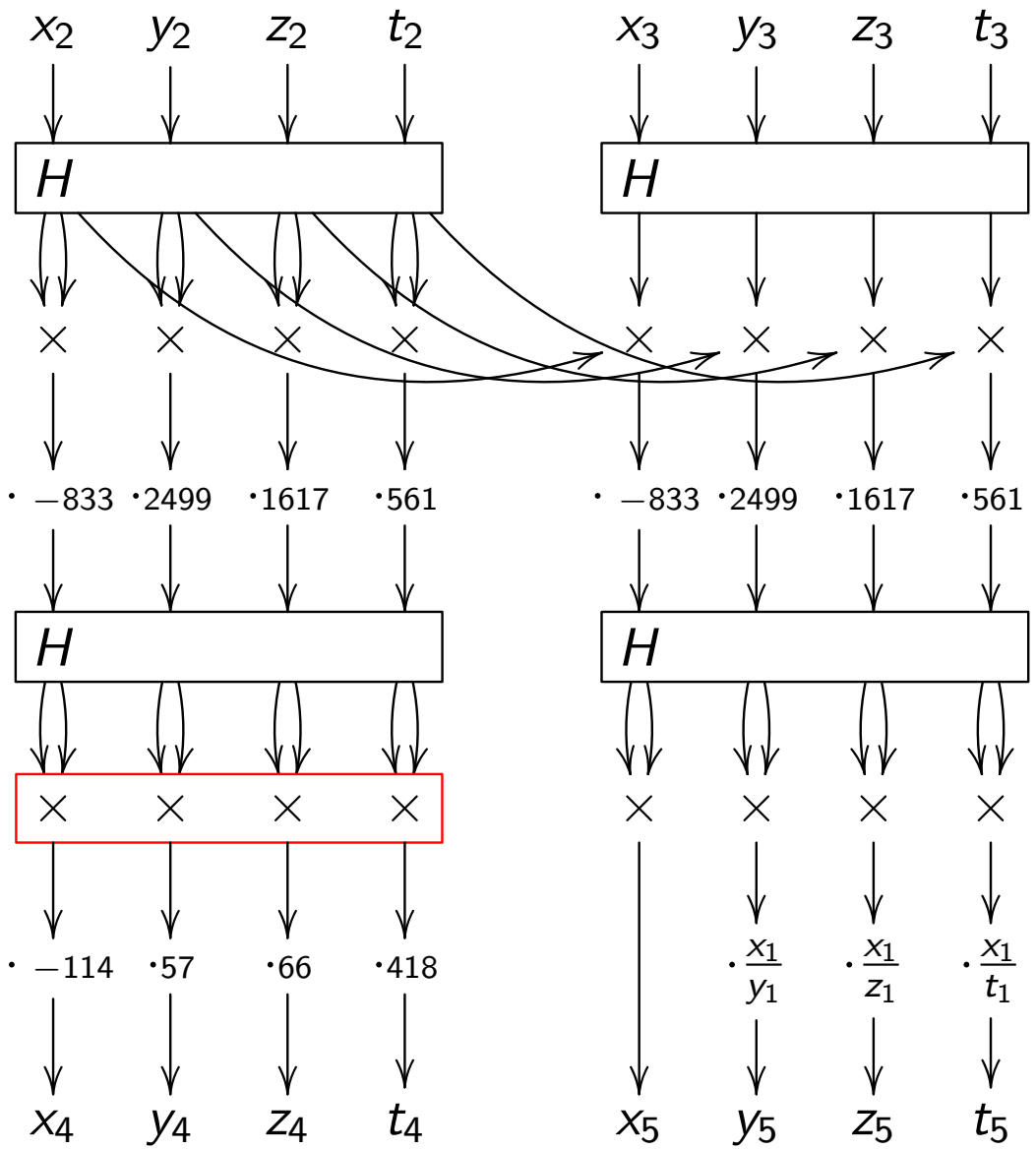

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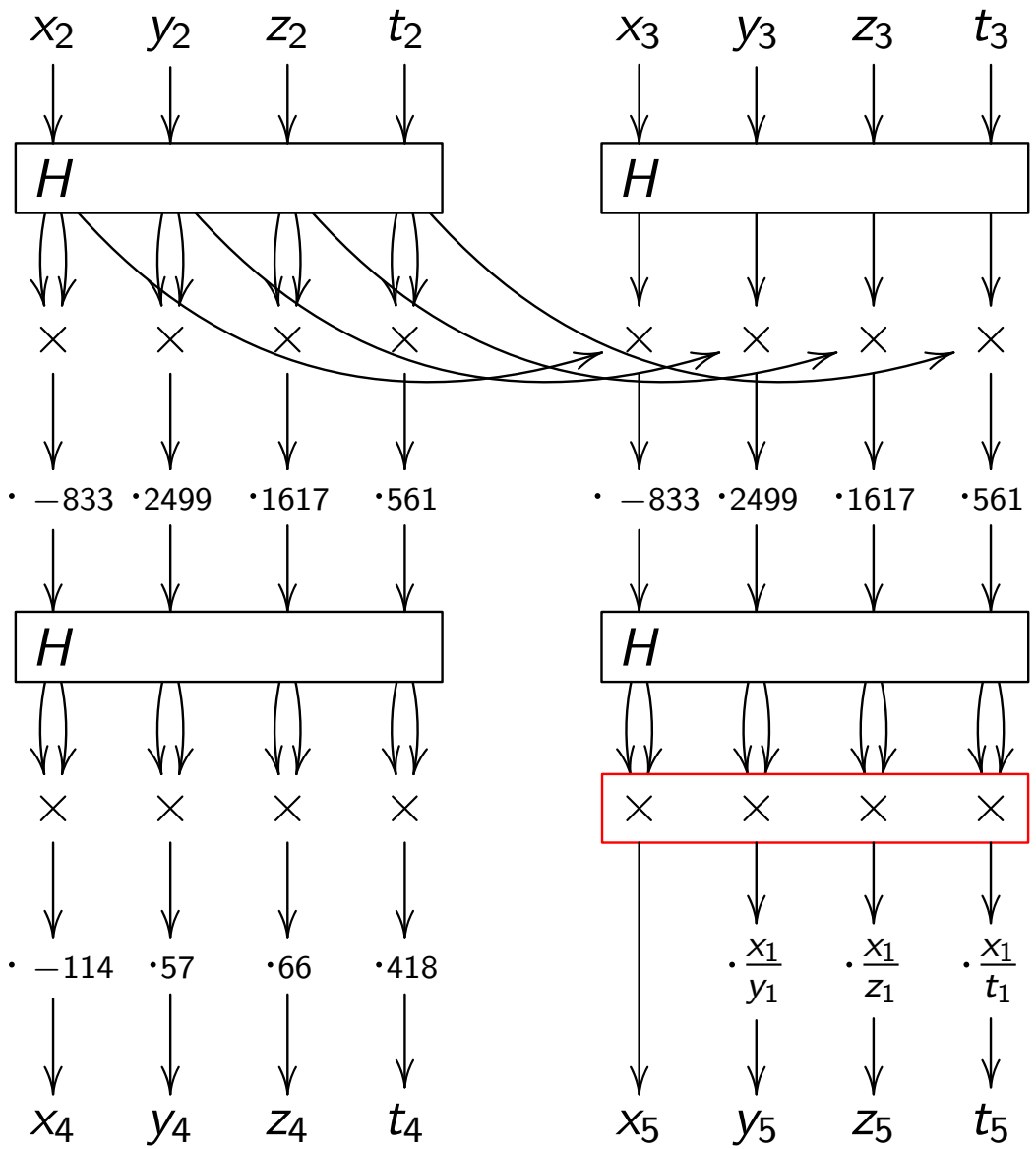

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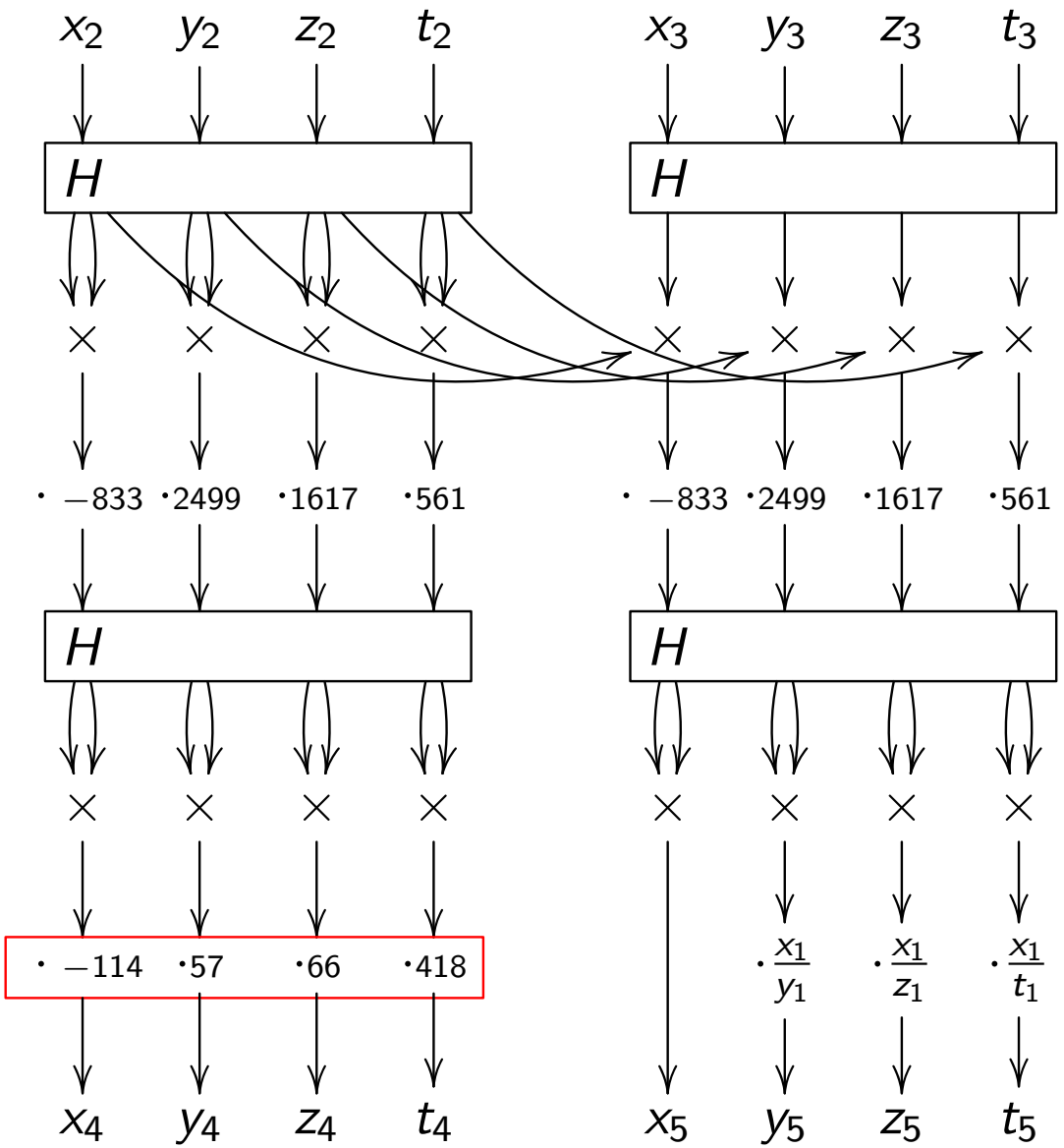

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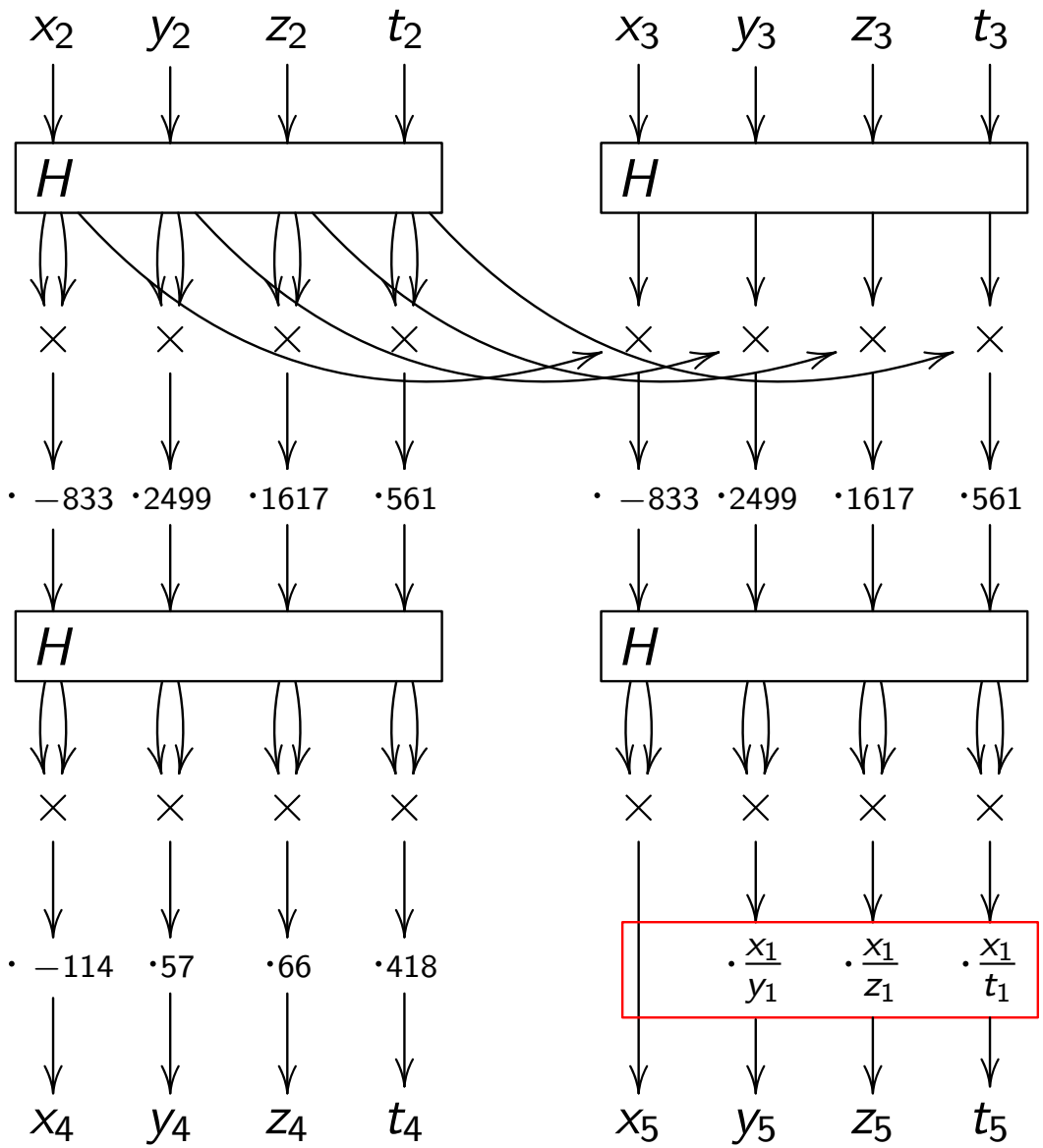

Making It Run Really Fast

- maximize usage of available vector multipliers
- minimize cost from carries
  - use redundant representation
  - use non-integer radix, e.g., $2^{25.4}$ on Cortex-A8
  - do not perform full carry
  - do parallel carry chain
- eliminate redundancy inside field operations
  - precompute to reuse values $2f_1, 2f_2, 2f_3, 2f_4$
- minimize overhead from permutations
  - organize data to fit instruction format
- minimize permutations in H
  - use different order of input/output and change sign
  - e.g. we reduced 144 Sandy Bridge permutations to 36
- schedule instructions to keep CPU as busy as possible
- see paper for details

Performance Comparison

Arch	Cycles	g	Field	Source of software
A8-slow	497389	1	$2^{255}-19$	BeSc CHES 2012
A8-slow	305395	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014
A8-fast	460200	1	$2^{255}-19$	BeSc CHES 2012
A8-fast	273349	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014
Sandy	194036	1	$2^{255}-19$	BeDuLaScYa CHES 2011
Sandy	153000?	1	$2^{252}-2^{232}-1$	Hamburg
Sandy	137000?	1	$(2^{127}-5997)^2$	LoSi Asiacrypt 2012
Sandy	122716	2	$2^{127}-1$	BoCoHiLa Eurocrypt 2013
Sandy	119904	1	2^{254}	OlLóArRo CHES 2013
Sandy	96000?	1	$(2^{127}-5997)^2$	FaLoSá CT-RSA 2014
Sandy	92000?	1	$(2^{127}-5997)^2$	FaLoSá July 2014
Sandy	88916	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014
Haswell	161648	1	$2^{255}-19$	BeDuLaScYa CHES 2011
Haswell	110740	2	$2^{127}-1$	BoCoHiLa Eurocrypt 2013
Haswell	61712	1	2^{254}	OlLóArRo CHES 2013
Haswell	60556	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014

Part II

Curve41417: Karatsuba revisited
CHES 2014

Joint work with Daniel J. Bernstein and Tanja Lange

Performance Budget

- This part focuses on ARM Cortex-A8

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- OpenSSL `secp160r1` (security level only 2^{80})
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- Kummer (security level 2^{128})
  - $\approx$ 0.3 million cycles on A8-fast and A8-slow
- Curve41417 (security level above 2^{200})
  - $\approx$ 1.6 million cycles on A8-fast
  - $\approx$ 1.8 million cycles on A8-slow

Design of Curve41417

- High-security elliptic curve (security level above 2^{200})
- Defined over prime field $\mathbf{F}_p$ where $p = 2^{414} - 17$
- In Edwards curve form

$$x^2 + y^2 = 1 + 3617x^2y^2$$

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- Large prime-order subgroup (cofactor 8)
- IEEE P1363 criteria (large embedding degree, etc.)
- Twist secure, i.e., twist of Curve41417 also secure

ECC Arithmetic

- Mix coordinate systems:
  - doubling: projective X, Y, Z
  - addition: extended X, Y, Z, T

  (See https://hyperelliptic.org/EFD/)
- Scalar multiplication:
  - signed fixed windows of width $\omega = 5$
  - precompute $0P, 1P, 2P, \ldots, 16P$
    also multiply $d = 3617$ to T coordinate
  - special first doubling
  - compute T only before addition

Double-and-add

Input: $Q, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R = nQ$

$R \leftarrow 1Q$

for $c = n_{i-2}$ **to** n_0
  $R \leftarrow 2R$
  if $c = 1$ **then**
    $R \leftarrow R + Q$

Return R

Example: Compute $9Q$

$9 = 1001_2$

$R = 1Q$

$n_2 = 0: \quad 2Q$

$n_1 = 0: \quad 4Q$

$n_0 = 1: \quad 8Q + Q = 9Q$

Windowing Method

Input: $Q, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R = nQ$

$R \leftarrow (n_{i-1}, \ldots, n_{i-k})_2$

for $c = \lfloor (i-1)/\omega \rfloor$ **down to** 0
  $R \leftarrow 2^\omega R + S_c$

Return R

Note: ω = window width

Example: Compute $2345Q$

$2345 = \underline{10}\ \underline{01001}\ \underline{01001}_2$

$R = 10_2 = 2Q$

$01001_2 = 9$:

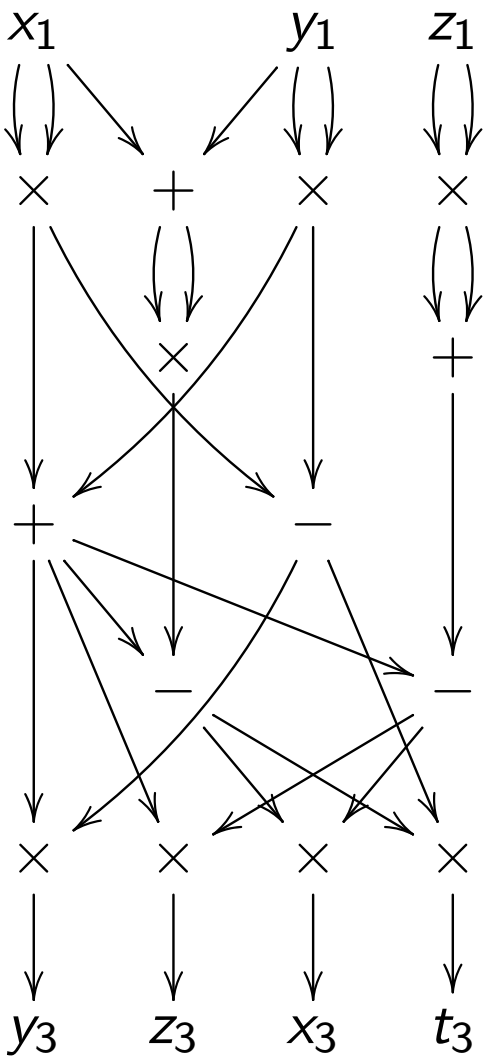
$2^5(2Q) + 9Q = 73Q$

$01001_2 = 9$:

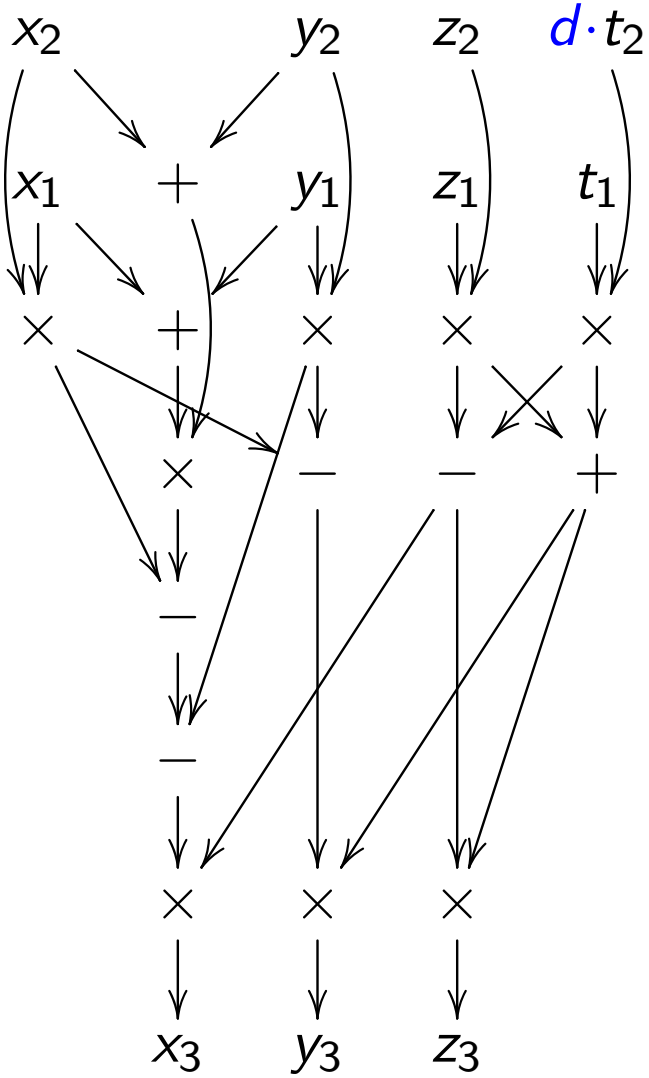
$2^5(72Q) + 9Q = 2345Q$

Point Operations

Point Doubling



Point Addition



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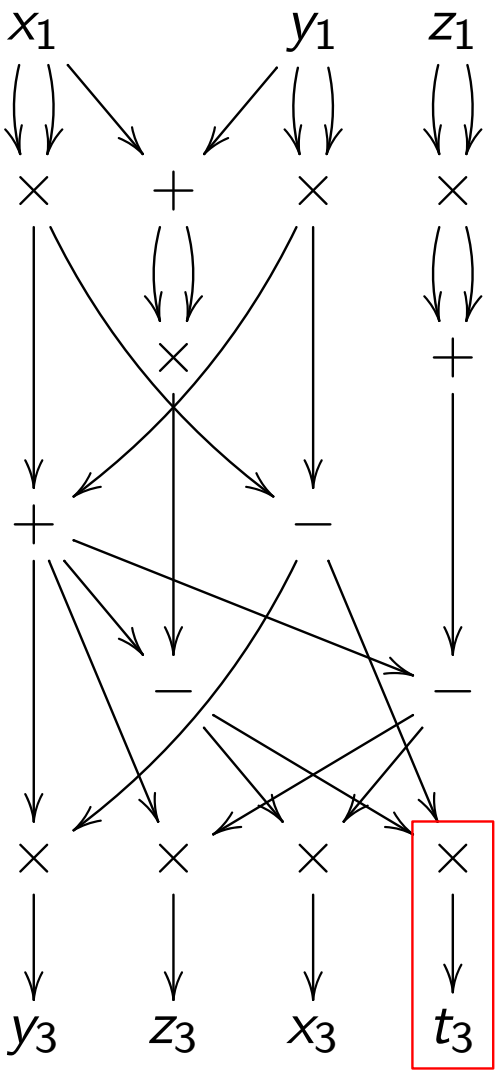

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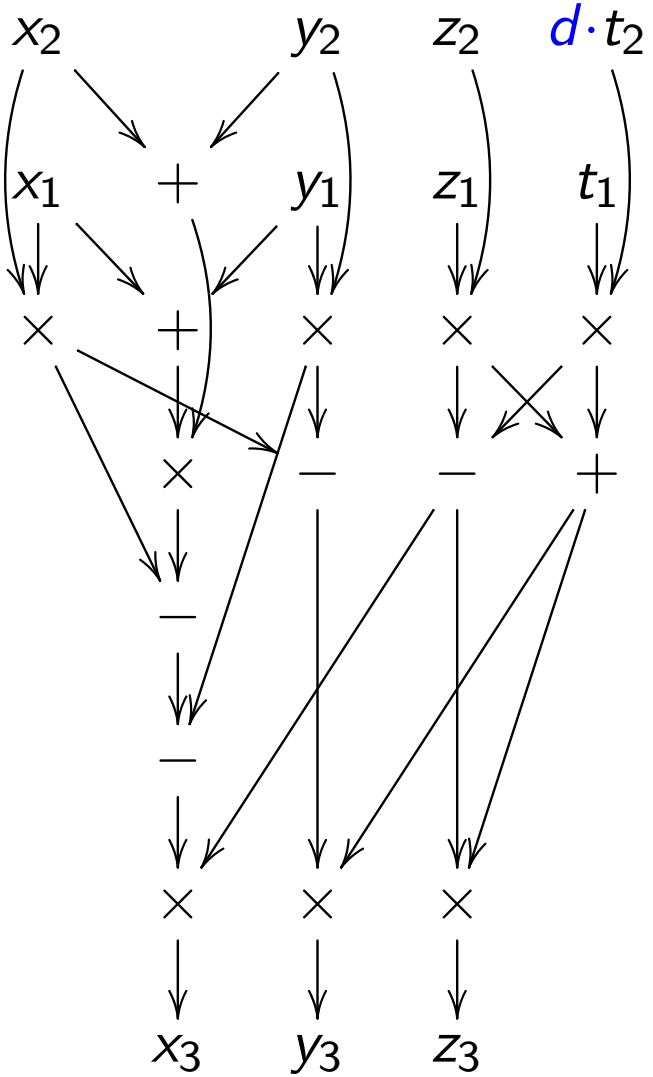

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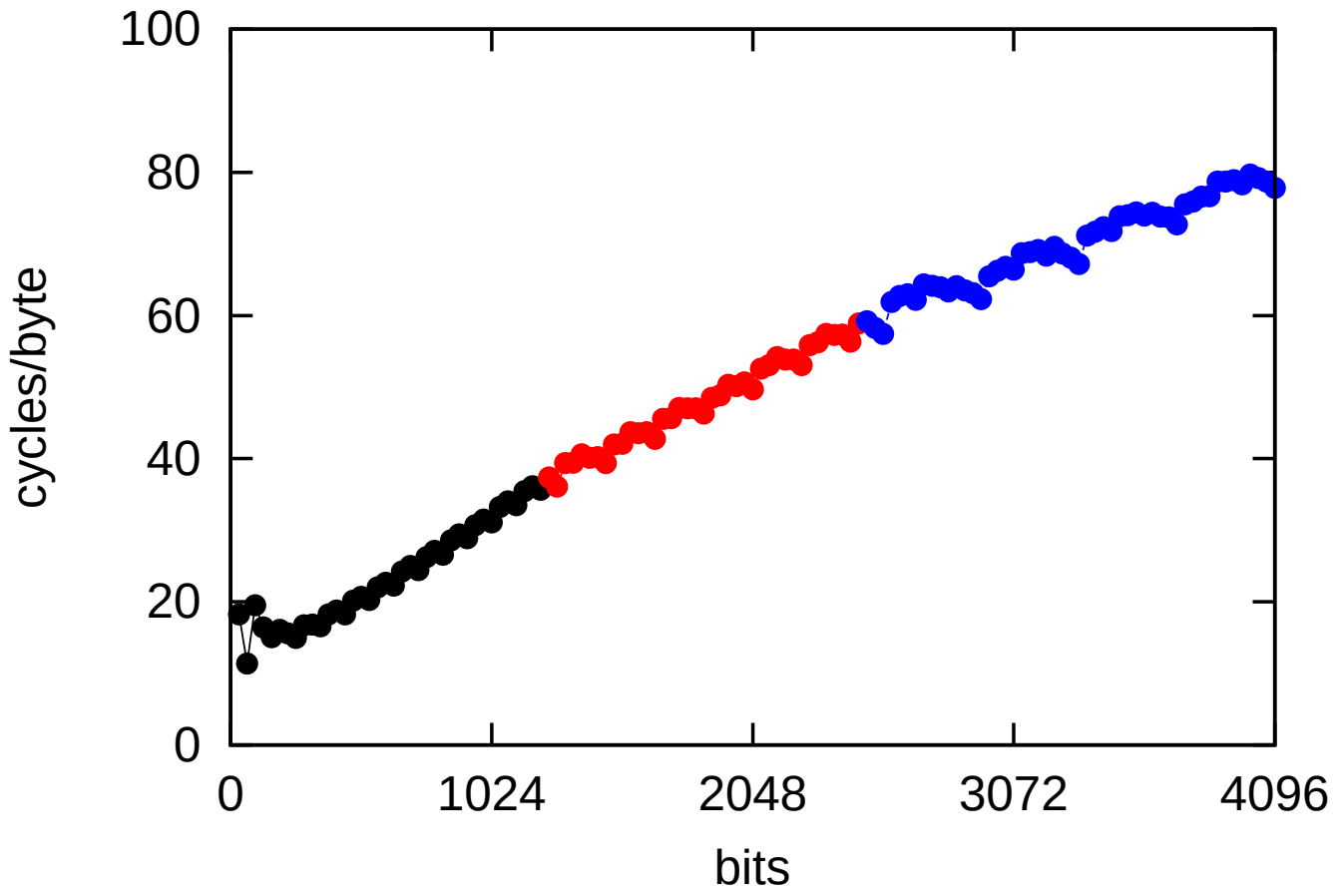
- What is the zero-level/one-level cutoff for number of limbs?

GMP's Cutoffs for Karatsuba



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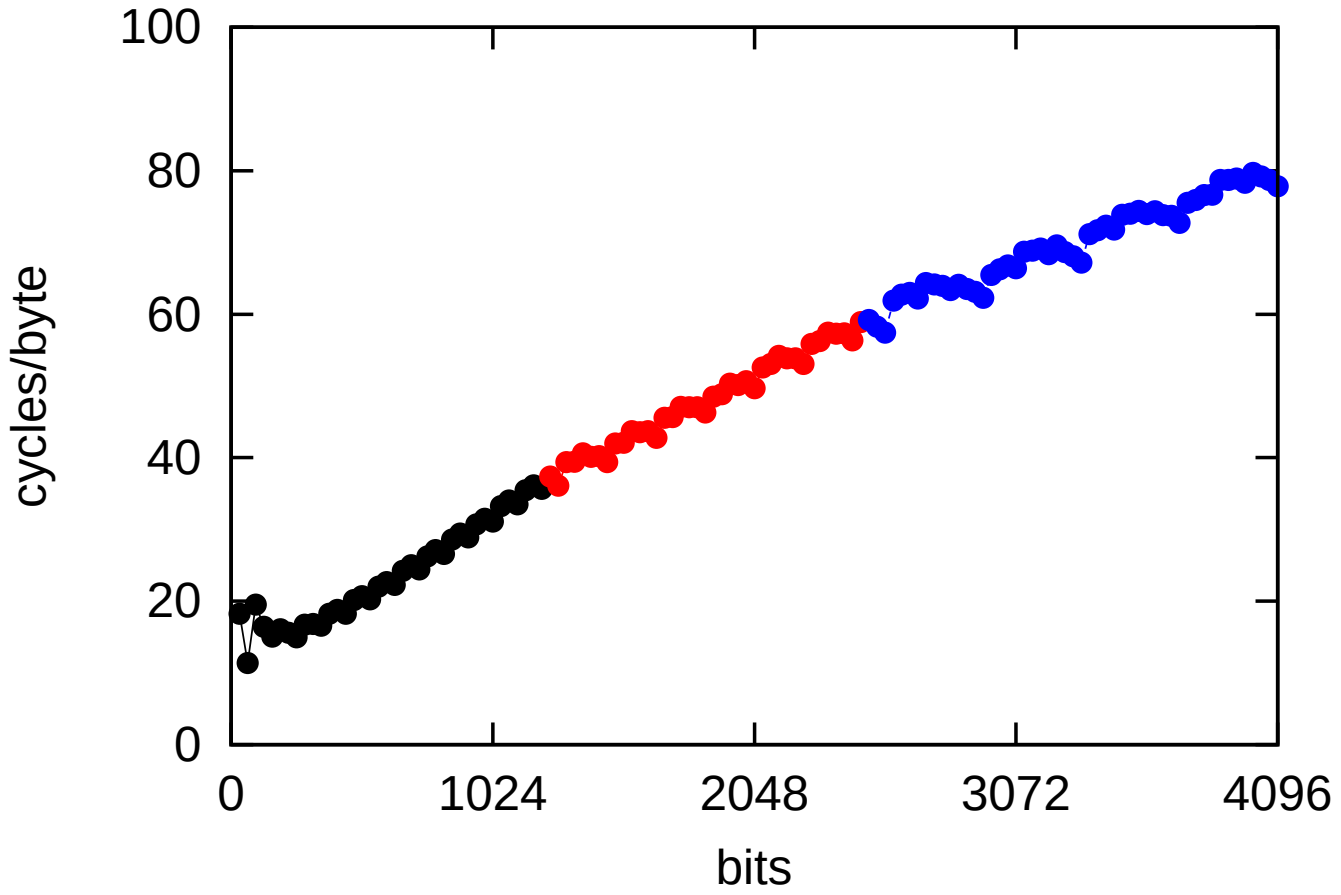

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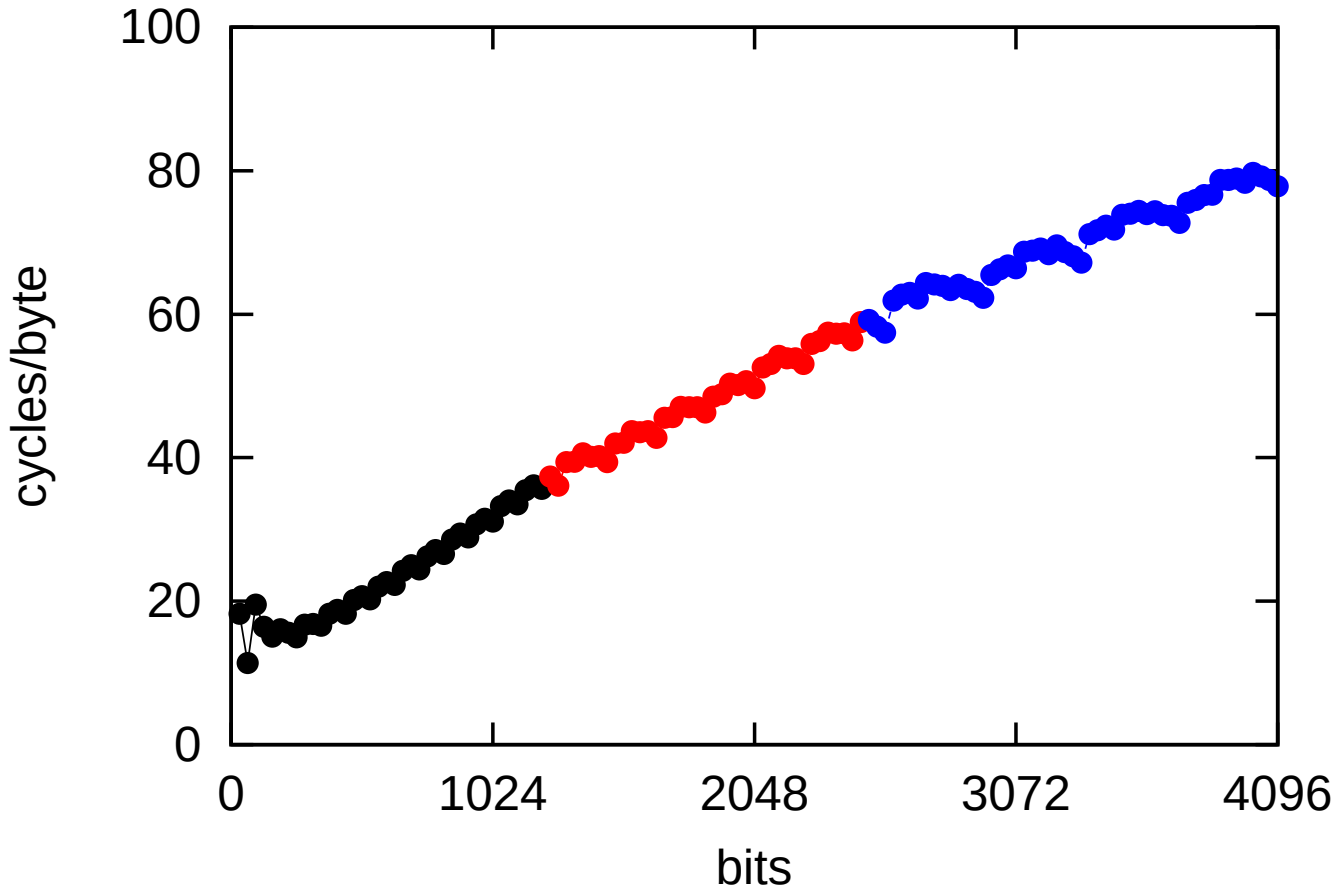

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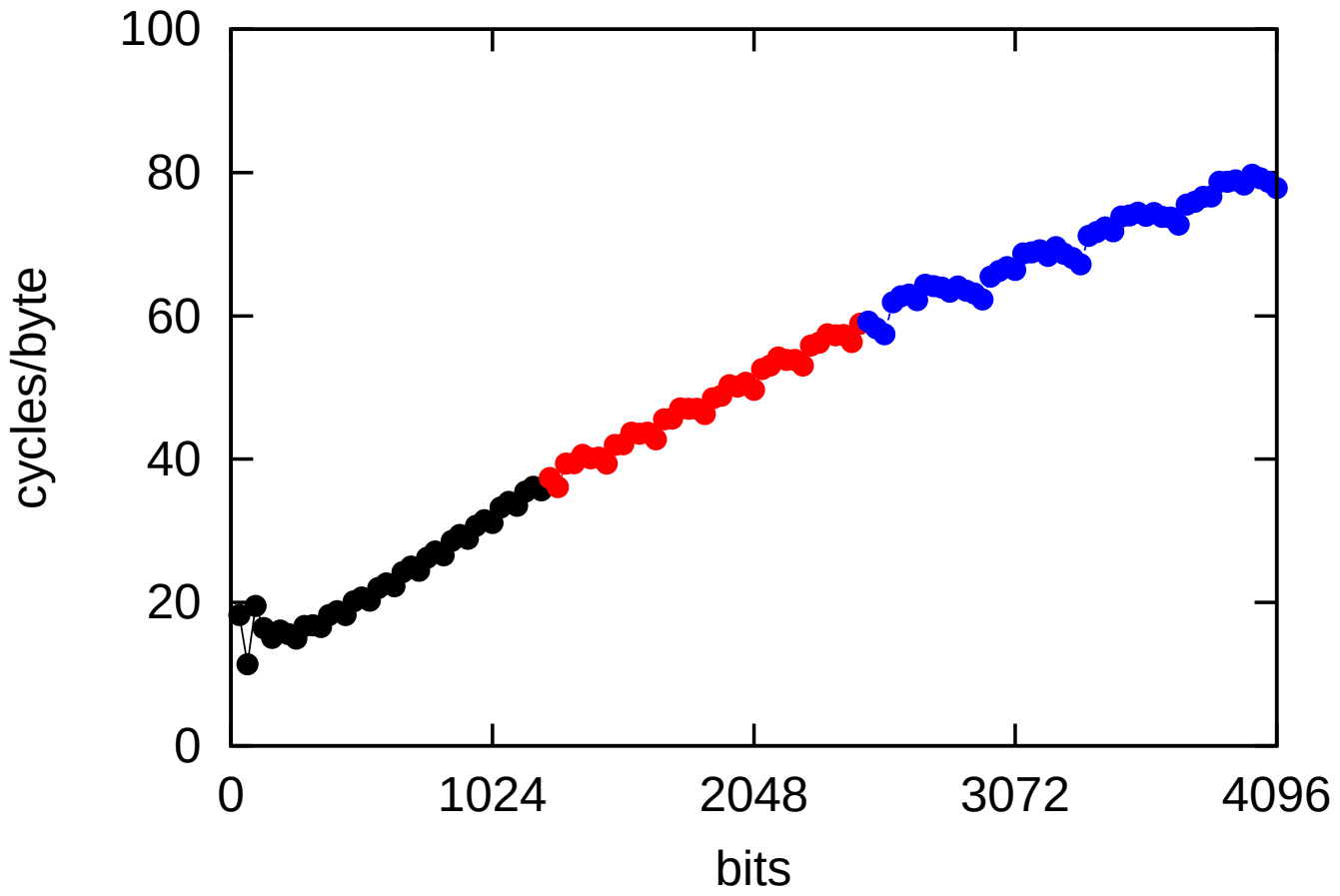

- GMP 6.0.0a library chooses 1248 bits on ARM Cortex-A8
- We reduce cutoff via improvements to Karatsuba
- We reduce cutoff via redundant representation

Polynomial Multiplication

- Goal: Compute $P = AB$
  given $A = a_0 + a_1 t^n$ and $B = b_0 + b_1 t^n$
- Method 1: schoolbook
  $P = a_0 b_0 + (a_0 b_1 + a_1 b_0) t^n + a_1 b_1 t^{2n}$
- Method 2: Karatsuba ($8n-4$ additions)
  $P = a_0 b_0 + ((a_0 + a_1)(b_0 + b_1) - a_0 b_0 - a_1 b_1) t^n + a_1 b_1 t^{2n}$
- Method 3: refined Karatsuba ($7n-3$ additions)
  $P = (a_0 b_0 - a_1 b_1 t^n)(1 - t^n) + (a_0 + a_1)(b_0 + b_1) t^n$

Polynomial Multiplication $\bmod Q$

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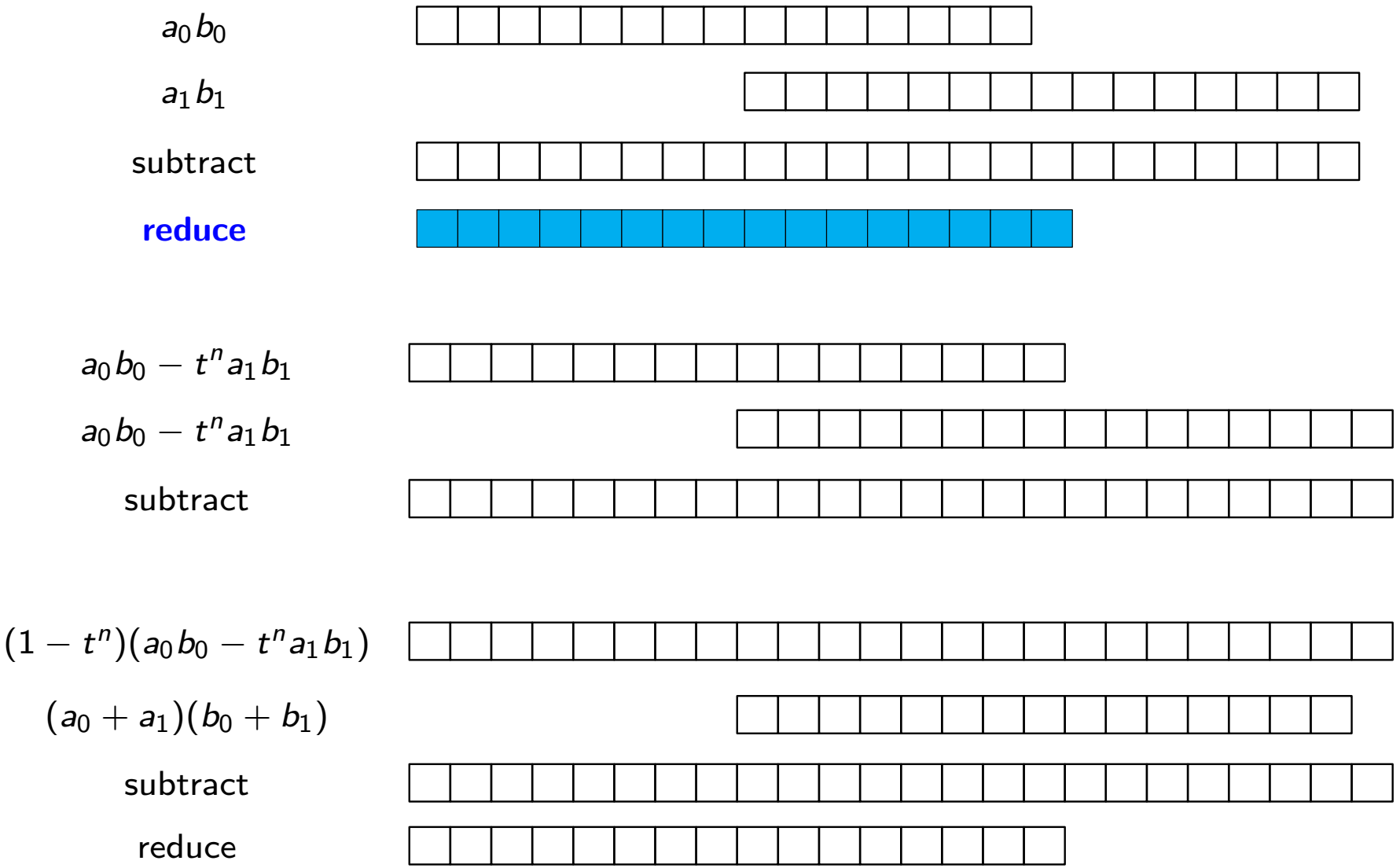
- Method 4: reduced refined Karatsuba ($6n-2$ additions) (new)
  $P = (a_0 b_0 - a_1 b_1 t^n \bmod Q)(1 - t^n) + (a_0 + a_1)(b_0 + b_1) t^n \bmod Q$

Reduced Refined Karatsuba



Cost Comparison (Karatsuba)

Level	Mult.	Add		Cost
		64-bit	32-bit	
0-level	256	15	0	$256+\ 8+\ 0 = 264$
1-level	192	59	16	$192+30+\ 4 = 226$
2-level	144	119	40	$144+60+10 = 214$
3-level	108	191	76	$108+96+19 = 223$

Note: use multiply-add instructions

Recall:

- 1 cycle per multiplication
- 0.5 cycle per 64-bit addition
- 0.25 cycle per 32-bit addition

Cost Comparison (refined Karatsuba)

Level	Mult.	Add		Cost
		64-bit	32-bit	
0-level	256	15	0	$256+\ 8+\ 0 = 264$
1-level	192	52	16	$192+26+\ 4 = 222$
2-level	144	103	40	$144+52+10 = 206$
3-level	108	166	76	$108+83+19 = 210$

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2-level	144	96	40	$144+48+10 = 202$
3-level	108	159	76	$108+80+19 = 207$

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0.25 cycle per 32-bit addition

Performance Comparison

- OpenSSL

curve	# cycle on i.MX515	# cycle on Sitara
`secp160r1`	$\approx$ 2.1 million	$\approx$ 2.1 million
`nistp192`	$\approx$ 2.9 million	$\approx$ 2.8 million
`nistp224`	$\approx$ 4.0 million	$\approx$ 3.9 million
`nistp256`	$\approx$ 4.0 million	$\approx$ 3.9 million
`nistp384`	$\approx$ 13.3 million	$\approx$ 13.2 million
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  - $\approx$ 1.6 million cycles on A8-fast (FreeScale i.MX515)
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- Coming soon: Intel Haswell implementation with Sebastian Verschoor